

# Towards a statistical paradigm for climate change

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**ABSTRACT:** It is argued that an acceptable 'paradigm' needs to be devised to clarify climate change as a statistical concept. A relatively simple statistical model for climate change is proposed as a start towards producing such a paradigm. Climate change is envisioned to involve changes in both the location and scale parameters of the probability distribution of a climate variable, whereas the shape of the distribution remains the same. Modifications of the model are treated to circumvent some apparent limitations of this approach, for instance, to allow for changes in the shape of the distribution of total precipitation. An indirect validation of the model is attempted through consideration of 2 analogues for global climate change: (1) a 'spatial analogue', involving patterns of variation in climate across space; and (2) the urban heat island, involving real temporal changes in local climate. These analogues indicate that any statistical model for climate change would need to be at least as complex as the one proposed here. Related issues include the need to preserve internal consistency when generating scenarios of climate change, as well as the need for information about how the variability of climate would change.

## INTRODUCTION

Interest in the statistical nature of climate change has heightened in recent years (e.g. Houghton et al. 1990). But attempts to characterize climate change in statistical terms have typically proceeded in an ad hoc fashion. Often the hypotheses tested or the assumptions made have not been explicitly stated. For instance, in testing for a trend over time in average climate, it has usually been tacitly assumed that the variability about the hypothetical trend line remains constant (e.g. Solow 1987, Bloomfield 1992). Likewise, projections of how the frequency of extreme events would change under the enhanced greenhouse effect have been based on the same possibly unrealistic assumption about no change in variance (Hansen et al. 1988).

Such approaches help to explain the current state of confusion, in which results concerning statistical changes in climate are conflicting and inconsistent (Solow & Broadus 1989). In this paper, it is argued that

little progress can be expected to be made in resolving these conflicts, unless an acceptable 'paradigm' is devised to clarify climate change as a statistical concept. Just as the need for a paradigm to monitor climate change has been recognized (Wood 1990), a model that defines climate change in statistical terms is required.

A relatively simple statistical model for climate change is proposed as a start towards producing the needed paradigm. Of course, it is recognized that a model does not, in and of itself, constitute a paradigm and that any model is necessarily open to criticism. Rather, the model is intended to provoke a change in the way of thinking about climate change. This particular model assumes that a climate variable possesses a probability distribution (possibly skewed in shape) with both a location and a scale parameter. Climate change is envisioned to involve changes in both of these parameters, whereas the shape of the distribution is assumed to remain unchanged.

Nevertheless, some limitations to this concept are apparent. For example, with the climate variable of total precipitation, a change in the shape of the probability distribution would naturally be expected. It will be demonstrated that this type of change can be incor-

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porated within the present framework by dealing directly with the underlying precipitation process.

The question of how to validate any statistical model for climate change is difficult to address. Two analogues are proposed in which, although the forcing factors are not directly related to greenhouse gases, at least it is clear that a real change in climate is involved. The 'spatial analogue' substitutes actual differences across space for hypothetical changes over future time horizons. The urban heat island consists of actual climate changes over time that are local, rather than global, in extent.

Attempting to come to grips with a statistical paradigm for climate change helps to clarify several related issues. The need to maintain internal consistency when generating scenarios of future climate is one such consideration. Another is the neglect of variability and the frequency of extreme events in research on climate change (already mentioned by Katz 1992).

## MODEL

### Location and scale parameters

Let the climate variable  $X$  have a probability distribution, with distribution function  $F(x) = \Pr\{X \leq x\}$ . This random variable  $X$  is said to possess a location parameter  $\mu$  and a scale parameter  $\sigma$ ,  $\sigma > 0$ , if the distribution of the standardized variable

$$Z = (X - \mu)/\sigma \quad (1)$$

does not depend on either  $\mu$  or  $\sigma$ . In the special case of  $F$  being the normal distribution, the location parameter  $\mu$  is simply the mean and the scale parameter  $\sigma$  is simply the standard deviation. Although climatologists may be unfamiliar with the more abstract concepts of location and scale, these parameters are more meaningful statistically than the mean and standard deviation when dealing with possibly non-normal distributions. Many distributions that are commonly fit to climate variables (e.g. exponential, extreme value, gamma, log normal, squared normal, Weibull) can be seen to fall within this framework, in some instances through use of transformations or approximations (Essenwanger 1976).

Climate change is envisioned to involve a combination of 2 different statistical operations: (1) the distribution function  $F$  is shifted, producing a change in location  $\mu$ ; and (2)  $F$  is rescaled, producing a change in scale  $\sigma$ . The first operation can be interpreted as an additive effect, creating a new random variable,  $X^*$  say, related (*in distribution*) to the original random variable  $X$  by

$$X^* = X + \Delta. \quad (2)$$

Here  $\Delta$  is some constant representing the difference in location parameter (i.e.  $\Delta = 0$  corresponds to no change in location). Specifically, the new variable  $X^*$  has location parameter  $\mu^* = \mu + \Delta$  and scale parameter  $\sigma^* = \sigma$ , but still the same form of distribution function  $F$ .

The second operation can be interpreted as a multiplicative effect, creating a new random variable,  $X^{**}$  say, related (*in distribution*) to  $X$  by

$$X^{**} = \mu + \delta(X - \mu). \quad (3)$$

Here  $\delta > 0$  is some constant representing the proportionate change in scale parameter (i.e.  $\delta = 1$  corresponds to no change in scale parameter). Specifically, the new variable  $X^{**}$  has location parameter  $\mu^{**} = \mu$  and scale parameter  $\sigma^{**} = \delta\sigma$ , but still the same form of distribution function  $F$ .

## Example

Fig. 1 illustrates this concept for one choice of distribution function,

$$F(x) = \exp[-e^{-(x-\mu)/\sigma}], \quad -\infty < x < \infty, \quad (4)$$

the Type I extreme value (or Gumbel) distribution (Johnson & Kotz 1970, Chapter 21), commonly used to model climate extremes (Tiago de Oliveira 1986). It is clear that  $\mu$  and  $\sigma$  in Eq. (4) do play the role of location and scale parameters (as in Eq. 1). However, the mean of  $X$  depends on both  $\mu$  and  $\sigma$ ,

$$E(X) \approx \mu + 0.577\sigma. \quad (5)$$

In fact, it turns out that the location parameter  $\mu$  is the mode of  $X$ . On the other hand, the variance of  $X$  is

$$\text{Var}(X) \approx 1.645\sigma^2, \quad (6)$$

only depending on the scale parameter  $\sigma$ .

The 2 hypothetical forms of climate change are included in Fig. 1: (i) changes in the location parameter  $\mu$  to 2 new values  $\mu^*$ , one higher and one lower, through the operation (2) with  $\Delta = \pm\sigma$ ; and (ii) changes in the scale parameter  $\sigma$  to 2 new values  $\sigma^{**}$ , one higher and one lower, through the operation (3) with  $\delta = 2$  and  $1/2$ . Although the shapes of all the distributions that appear in Fig. 1 are identical [i.e. all being the Type I extreme value distribution (Eq. 4)], a wide range of changes in climate can be achieved, especially if both of the operations, (2) and (3), are applied simultaneously.

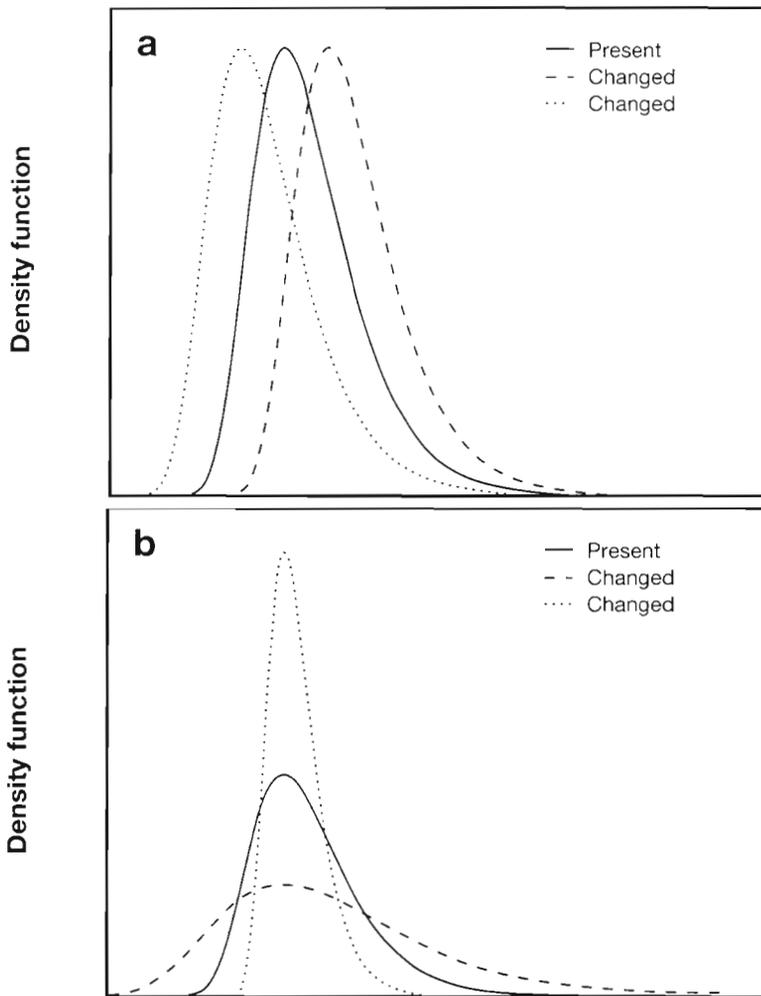


Fig. 1. Hypothetical changes in distribution of climate variable with location parameter  $\mu$  and scale parameter  $\sigma$ : (a) for 2 new values  $\mu^*$  of  $\mu$ , holding  $\sigma$  constant; and (b) for 2 new values  $\sigma^{**}$  of  $\sigma$ , holding  $\mu$  constant. Solid curve shows present distribution, whereas dashed and dotted curves represent new distributions

## MODIFICATIONS

In contrast with this particular statistical model for climate change, so far most scenarios of future climate have been generated under the assumption that only the location parameter  $\mu$  changes, not the scale parameter  $\sigma$ . For instance, Hansen et al. (1988) examined how the frequency of certain extreme temperature events would change with increases in the atmospheric concentrations of greenhouse gases. They did so by only allowing the mean temperature to change on the basis of the outcomes of general circulation model (GCM) experiments. The Intergovernmental Panel on Climate Change did mention the possibility of both location and scale changes (Houghton et al. 1990, p. 152), but their scientific assessment did not reflect any systematic adoption of this statistical model.

It should be clear that the relative frequency of extreme events is quite dependent on any changes in scale, not just changes in location (Mearns et al. 1984, Katz & Brown 1992). In fact, it would be more plausible to argue that this location and scale parameter model (Eq. 1) for climate change is not general enough. In particular, is it justifiable to not allow for a possible change in the shape of the distribution? Although changes in shape certainly cannot necessarily be ruled out, this issue can be circumvented in some cases by applying the basic model in the most appropriate manner.

## Precipitation example

As is well established empirically, the shape of the distribution of precipitation totaled over some time period (e.g. a month or season) varies spatially, with a marked tendency to be more positively skewed the drier the climate (e.g. Waggoner 1989). Similar characteristics are evident when comparisons are made among seasons for the same site. This behavior can be explained by means of a probabilistic argument to be outlined in this section, providing a chance mechanism on whose basis the proposed location and scale parameter model (Eq. 1) for climate change can still be utilized.

A characteristic of the precipitation process that is crucial to any stochastic theory is its being the composition of 2 component processes: one consisting of the sequence of occurrences (or non-occurrences) of precipitation, and the other being the sequence of intensities of precipitation (i.e. amounts when it does occur). This decomposition implies that the total precipitation over some time period of length  $T$ , denoted by  $S_T$  say, can be represented probabilistically as a so-called 'random sum'. That is,

$$S_T = X_1 + X_2 + \dots + X_N, \quad (7)$$

where the random variable  $N$  is the total number of occurrences of precipitation within the time period and  $X_i > 0$  is the intensity corresponding to the  $i^{\text{th}}$  occurrence,  $i = 1, 2, \dots, N$ .

Any climate change should, at a minimum, allow for both a change in the frequency of occurrence of pre-

precipitation and a change in the scale parameter of the distribution of precipitation intensities. On the other hand, the intensities might be assumed to retain the same shape of distribution (these exact assumptions have sometimes been made about the effects of cloud seeding to modify precipitation; Crow 1978). Such seemingly innocuous changes in the parameters of the precipitation process would induce more complex changes in the distribution of total precipitation. By the Central Limit Theorem for random sums (Feller 1971), a change in the frequency of occurrence of precipitation alone would produce a change in the shape of the distribution of total precipitation (either more or less similar to the normal), in spite of the fact that the shape of the distribution of precipitation intensities remains the same.

This effect on the distributional shape of total precipitation can be quantified by employing a stochastic model for precipitation, say on a daily time scale – for instance, one known as a ‘chain-dependent process’ (Todorovic & Woolhiser 1975, Katz 1977). The daily occurrence process is modeled as a first-order Markov chain, with parameters  $\pi$  being the probability of a wet day and  $\rho$  the first-order autocorrelation coefficient (or ‘persistence parameter’). In particular, the expected number of wet days within a time period of length  $T$  days is given by

$$E(N) = T\pi. \quad (8)$$

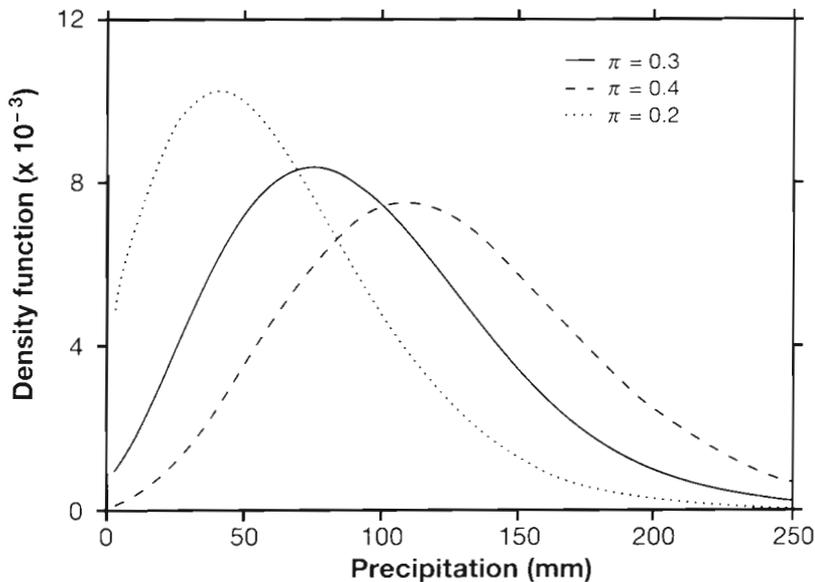


Fig. 2. Hypothetical distribution of January total precipitation at Chico, California, USA, for 3 different values of the probability of a wet day  $\pi$ , holding the other parameters of the chain-dependent model for daily precipitation constant. Solid curve for present climate of  $\pi = 0.3$ , dashed curve for  $\pi = 0.4$ , and dotted curve for  $\pi = 0.2$ .

The daily intensity process consists of independent and identically distributed positive-valued random variables. For simplicity, this common distribution is taken to be the exponential, with

$$\Pr\{X_i \leq x\} = 1 - e^{-\lambda x}, \quad \lambda, x > 0, \quad (9)$$

and the mean intensity being  $E(X_i) = 1/\lambda$ .

Fig. 2 shows the effects on the distribution of total precipitation  $S_T$  when only the probability of a wet day  $\pi$  is changed (i.e. the other 2 daily parameters,  $\rho$  and  $\lambda$ , are held constant). The current climate is chosen to mimic the daily precipitation statistics for the month of January at Chico, California, USA (Katz & Parlange 1993), with  $\pi = 0.3$ ,  $\rho = 0.4$ , and  $\lambda = 0.1 \text{ mm}^{-1}$ . The exact distribution of January total precipitation (i.e.  $T = 31 \text{ d}$ ) is calculated for the present climate, as well as for both increases and decreases in  $\pi$  (i.e.  $\pi = 0.4$  and  $0.2$ ). In addition to the anticipated effects of higher mean and variance of monthly total precipitation with higher  $\pi$ , it is evident that the degree of skewness of this distribution decreases as  $\pi$  increases (consistent with the aforementioned Central Limit Theorem for random sums).

Waggoner (1989) considered the relationships among the mean, variance, and shape of the distribution of annual total precipitation, and Katz & Garrido (unpubl.) quantified the relative sensitivity of extreme precipitation events to changes in the median or in the scale parameter of the distribution of monthly or seasonal total precipitation. But these approaches were unable to identify the mechanism by which the underlying precipitation process could generate such changes. Moreover, the evidence is beginning to accumulate in GCM experiments for differential changes in the occurrence and intensity processes due to the enhanced greenhouse effect (Gordon et al. 1992). So it is especially important to adopt an approach, such as the one described here, that is capable of treating these component processes separately.

## VALIDATION

The issue of how best to validate the statistical model for climate change remains unclear. Certain types of observational analyses are fraught with difficulties in interpretation. The most popular idea to date has been to exam-

ine the climate record for the purpose of constructing historical analogies (Glantz 1988). For instance, an extended period of warmer or drier than normal conditions might be taken as analogous to some future time horizon under the enhanced greenhouse effect (e.g. the 'Dust Bowl' of the 1930s in the U.S. Great Plains) (Easterling et al. 1992). One obvious weakness of this approach is that past climate events may well represent transient features of an unchanging climate, rather than permanent changes. Searching for relationships among different parameters (e.g. means, variances, frequencies of extreme events) by this technique (e.g. Robinson 1992) is particularly problematic from a statistical perspective.

Suppose that when the sample mean and variance are repeatedly calculated for different time periods within the climate record, an apparent pattern emerges. For instance, periods of above-average temperature might be associated with lower-than-usual variability. It is well known that when drawing random samples from a normal distribution, the mean and variance of the samples are probabilistically independent. Thus, one would not expect to observe any relationship between the sample mean and sample variance for a stationary climate variable with a normal distribution. However, this property essentially characterizes the normal distribution (e.g. Johnson & Kotz 1970, Chapter 13). In general, sample statistics are themselves correlated, even if the climate time series were stationary. Consequently, it is difficult, if not impossible, to distinguish this purely statistical artifact from that of a non-stationary climate. What is really needed are analogues for which it is certain that a real climate change has taken place.

### Spatial analogue

If a change in the future climate were analogous to a spatial relocation (Waggoner 1991), then the climate variable under consideration should have the same form of distribution at each point within the region, except for an adjustment to allow for spatial differences in the location and scale parameters. Formally, let  $X_s$  denote the climate variable at site  $s$ , with location parameter  $\mu_s$  and scale parameter  $\sigma_s$ . Then the spatial analogue would be consistent with the proposed model

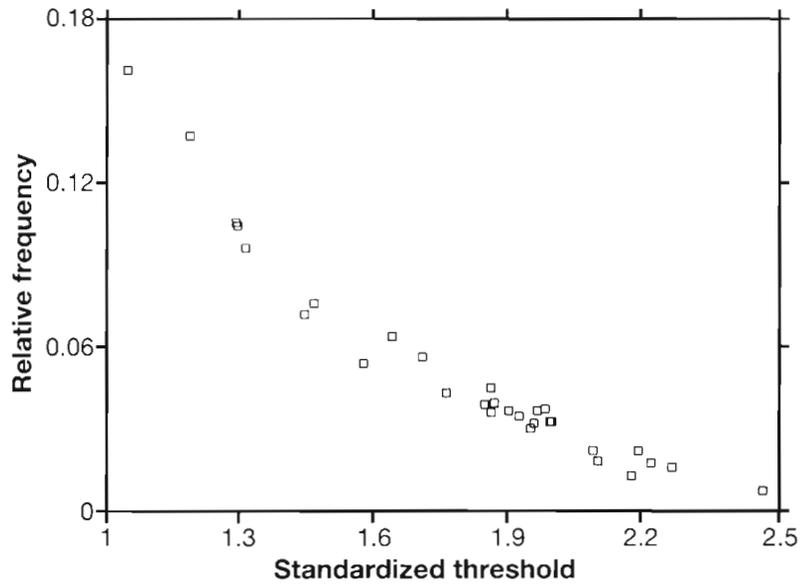


Fig. 3. Relative frequency of an extreme event, maximum temperature on a given day within July exceeding a threshold of  $c = 37.8$  °C, versus standardized threshold  $(c - \mu_s)/\sigma_s$  for 30 stations in the U.S. Midwest

for climate change (Eq. 1), if the distribution of the standardized quantity

$$Z_s = (X_s - \mu_s)/\sigma_s \quad (10)$$

were independent of the spatial index  $s$ .

Brown & Katz (1993) have studied whether extreme maximum and minimum daily temperature events have a spatial structure that satisfies this property (Eq. 10). They have found good agreement for maximum daily temperature in the summer within a portion of the U.S. Midwest and for minimum daily temperature in the winter within a portion of the U.S. Southeast. In other words, when adjustments for spatial differences in location and scale parameters are made, these temperature variables appear to have the same form of distribution. It should be noted that this type of regional analysis closely resembles hydrologic studies concerned with the regional estimation of flood probabilities (Hosking & Wallis 1993). For extreme precipitation events, hydrologists have verified that condition (10) holds to a good approximation for regions the size of water basins.

As a simple illustration, Fig. 3 shows the relative frequency of the maximum temperature on a given day within July exceeding a threshold of  $c = 37.8$  °C for the same 30 stations in the U.S. Midwest treated by Brown & Katz (1993). The scatterplot of these relative frequencies (i.e. estimates of  $\Pr\{X_s > c\}$ ) versus the standardized threshold  $(c - \mu_s)/\sigma_s$  indicates that these points do fall remarkably close to a smooth, decreasing

curve. This curve represents the right-hand tail of the assumed common form of distribution of maximum temperature across the region under condition (10). Such behavior is consistent with the proposed statistical model for climate change, with no explicit assumptions being made about the particular shape of the distribution (e.g. not necessarily the normal). Brown & Katz (unpubl.) found similar behavior for other maximum temperature thresholds, as well as for extreme minimum temperature events.

### Heat island effect

As metropolitan areas have developed over recent centuries, they have inadvertently produced a warming directly attributable to the human modification of the landscape (e.g. Landsberg 1981). For sites that have endured rapid growth, this warming is comparable in magnitude to that anticipated to have occurred so far as a result of the enhanced greenhouse effect (Changnon 1992). Moreover, this effect has been detected for cities across the globe, ranging from the tropics to high latitudes and to a lesser extent even in relatively small communities.

Research on the urban heat island has dwelt on average temperature effects, with little mention of any changes in variability or in the frequency of extreme events (e.g. very high maximum temperatures or very low minimum temperatures). One notable exception is the work by Balling et al. (1990). They examined the trend in the occurrence of extreme maximum and minimum temperatures at Phoenix, Arizona, USA, an area that has experienced a marked heat island effect in recent decades. Among other things, it was established that the statistical model for climate change in which simply the mean (or location parameter) is changed (e.g. as employed by Hansen et al. 1988) is not consistent with these observed changes in the frequency of extremes. Although changes in the mean are apparently sufficient to explain the trend in occurrence of extreme minimum temperatures, such a model overestimates the trend in frequency of extreme maximum temperatures.

Tarleton & Katz (1993) performed a re-analysis of the Phoenix temperature data. They allowed for changes in the standard deviation, along with the mean, of daily maximum temperature, taking the distribution to be the normal. Fig. 4 shows a comparison of how well

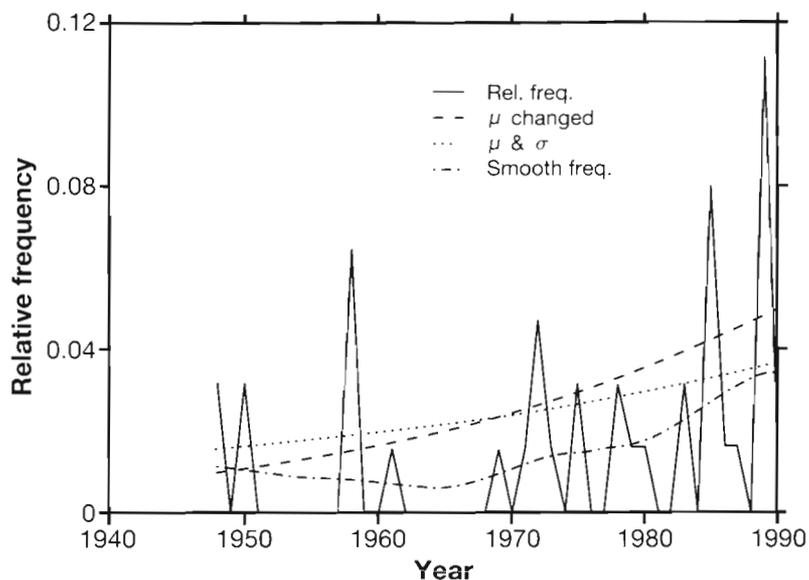


Fig. 4. Probability of extreme event, maximum temperature on a given day in July–August at Phoenix, Arizona, USA, exceeding a threshold of  $c = 45.3$  °C, for time period 1948–1990: the solid curve is observed relative frequency (the dot-dashed curve was smoothed using repeated hanning); and the dashed curve is estimates based on changing mean, but constant standard deviation (dotted curve for changing mean and standard deviation)

these 2 statistical models for climate change account for the observed pattern in the relative frequency of occurrence of the extreme event, the maximum temperature on a given day during July–August exceeding a threshold of  $c = 45.3$  °C. To remove the year to year fluctuations attributable to sampling errors, the time series of observed relative frequencies is smoothed using repeated hanning (i.e. a local smoother involving weights  $\frac{1}{4}$ ,  $\frac{1}{2}$  and  $\frac{1}{4}$ ; see Tukey 1977). It is evident that the extent of overestimation noted by Balling et al. (1990) is somewhat diminished when a trend in the standard deviation is permitted. The remaining overestimation is possibly attributable to the right-hand tail of the distribution being 'lighter' than the normal.

Phoenix is situated within a desert region, and the features of such an urban heat island may not be typical of those in other environments. Still it would be surprising if changes in temperature variability have not occurred as part of other heat islands. A more systematic study of this issue would be informative.

## RELATED ISSUES

### Internal consistency

Attention has been focused so far in this paper on the marginal (or unconditional) distributions of individual

climate variables. Of course, other important statistical features of climate include its temporal and spatial correlations, as well as cross correlations between different climate variables. When producing scenarios of future climate, these characteristics need to be taken into account to insure internal consistency. Inconsistent treatment of climate variables arises when certain parameters are varied and unintended changes are induced. These unanticipated changes might involve other statistics for the same variable or statistics for another related climate variable. Others have mentioned the need for internal consistency, but as a physical rather than a statistical concept (Wigley et al. 1986, Lamb 1987).

One way to automatically guarantee internal consistency for a single climate variable is to simply modify the existing historical time series (e.g. Mearns et al. 1992). For instance, operations (2) and (3) could be applied to adjust the mean and standard deviation. But this approach has the drawback that any future climate scenarios would be perfectly correlated with the historical record. Even if the climate were not changing, surely the future time series for the variable should be virtually uncorrelated with the past, just reflecting the natural variability of climate from year to year. One is driven towards the realization that a simulation approach is necessary in which a random component, representative of the natural variability of climate (e.g. on an annual time scale), is included.

#### AR(1) process

Internal consistency is relatively easy to guarantee in simulations when dealing with relative measures of dependence, such as autocorrelation or cross correlation coefficients. For instance, consider a single time series,  $X_t$  say,  $t = 1, 2, \dots$ , modeled by a first-order autoregressive [AR(1)] process,

$$X_t - \mu = \phi(X_{t-1} - \mu) + \varepsilon_t. \quad (11)$$

Here  $\mu$  is the mean,  $\phi$  is the first-order autocorrelation coefficient, and  $\varepsilon_t$  is the underlying, mean zero time series of uncorrelated, normally distributed errors (i.e. a 'white noise' process). The variance  $\sigma^2$  of the climate variable  $X_t$  is related to the variance of the underlying error term  $\varepsilon_t$ ,  $\sigma_\varepsilon^2$  say, by the relationship

$$\sigma_\varepsilon^2 = (1 - \phi^2)\sigma^2. \quad (12)$$

It is important to note that  $\mu$  and  $\sigma$  still play the role of location and scale parameters for the climate variable (as in Eq. 1).

If it is desired to change the process variance  $\sigma^2$  without changing the autocorrelation  $\phi$ , then the relationship (12) shows how the innovation variance  $\sigma_\varepsilon^2$  must be changed to preserve this internal consistency. In particular, Mearns et al. (1984) relied on this approach to generate time series of daily maximum temperature with differing mean, variance, and autocorrelation. Nevertheless, climate change simulations have been performed using this same AR(1) model (Eq. 11), with the autocorrelation parameter  $\phi$  being varied while the error variance  $\sigma_\varepsilon^2$  is held fixed (Bassett 1992). For this simulation exercise, Eq. (12) implies that the variance  $\sigma^2$  of the climate variable must be changing as  $\phi$  changes, a fact that might not be obvious to the uninitiated.

#### Richardson's model

In many applications (e.g. crop-climate models), simulations of the joint behavior of 2 or more climate variables, such as daily time series of temperature and precipitation, are required. The complex nature of the dependence among such time series may make it difficult to determine how to change the various parameters in an internally consistent manner. For instance, a model proposed by Richardson (1981) is expressed in a conditional form, convenient for simulating such joint time series of precipitation and temperature (i.e. the distribution of temperature is specified conditional on precipitation). It is much more complex to infer the marginal (or unconditional) properties of the individual time series for such a model. If care is not taken, unintended changes will be produced when certain parameters are varied. Wilks (1992) has demonstrated how this particular stochastic model can be adapted to generate scenarios of future climate change.

A simplified version of Richardson's model is briefly described. Let the random variable  $Y$  denote the maximum (or minimum) temperature on a given day. The conditional distribution of  $Y$  is taken as normal, with the mean and standard deviation depending on whether or not precipitation occurs on the same day. In particular,  $Y$  has one conditional mean given a dry day,  $\mu_Y(0)$  say, and another conditional mean given a wet day,  $\mu_Y(1)$  say. Now the unconditional (or overall) mean of  $Y$ ,  $\mu_Y$  say, is related to these 2 conditional means by

$$\mu_Y = (1 - \pi)\mu_Y(0) + \pi\mu_Y(1). \quad (13)$$

Here  $\pi$  denotes the probability of a wet day, as in the previous precipitation example. From Eq. (13), it is evident that simply changing the frequency of occurrence of precipitation will induce a change in the marginal distribution of temperature.

Table 1. Richardson's model for daily maximum temperature and precipitation occurrence during mid July at Columbia, Missouri, USA.  $\pi$ : probability of a wet day;  $\mu_Y(0)$ : mean max. temperature on a dry day;  $\mu_Y(1)$ : mean max. temperature on a wet day;  $\mu_Y$ : overall mean max. temperature

Climate	$\pi$	$\mu_Y(0)$ (°C)	$\mu_Y(1)$ (°C)	$\mu_Y$ (°C)
Current	0.31	33.1	31.0	32.4
Drier	0.21	33.1	31.0	32.7
Warmer	0.31	34.1	32.0	33.4
Drier, warmer	0.21	33.8	31.7	33.4

As a concrete example, Table 1 gives the parameter estimates for the current climate at Columbia, Missouri, USA, for daily maximum temperature and precipitation occurrence during mid July, the peak of the seasonal cycle for temperature (Richardson & Wright 1984). Not surprisingly, on the average, wet days have lower maximum temperatures than do dry days. A decrease in the probability of a wet day (i.e. from 0.31 to 0.21), without any change in the conditional mean maximum temperatures, still produces (by Eq. 13) an increase in the overall mean maximum temperature of about 0.3 °C. Because of this effect, to produce a 1 °C increase in the overall mean maximum temperature, the conditional mean maximum temperatures should only be increased by about 0.7 °C, *not* 1 °C (see Table 1).

Hutchinson (1986) reviewed methods for generating time series of climate variables, including Richardson's model. Other studies concerned with employing such methods to produce scenarios of future climate change include Reed (1986) and Woo (1992).

### Extremes/variability

Appreciation of the need for a statistical paradigm for climate change arises when considering how the relative frequency of extreme events might change as more conventional statistics, such as the mean or standard deviation, change. Upon reflection, it should be evident that simply changing the mean (or, more generally, the location parameter) could not be statistically defensible. The role of variability (more precisely, the scale parameter) is critical and, in fact, turns out to be relatively more important than that of the location parameter for extreme enough events (Katz & Brown 1992).

The question naturally arises as to why so little attention has been paid to possible changes in the variance/scale of climate. This lack of attention is evident, both in observational studies that attempt to detect

changes in climate and, to an even greater extent, in climate change experiments that make use of GCMs (Katz 1992). Clearly, information about changes in climate variability (and related extreme events) is critical to any assessments of the societal impacts of climate change (Parry & Carter 1985, Wigley 1985). This situation provides perhaps the most compelling evidence that a statistical paradigm for climate change is currently lacking.

### SUMMARY AND CONCLUSIONS

An attempt has been made to provoke a different way of thinking about climate change, clarifying the concept in statistical terms. Despite its obvious shortcomings, a statistical model for climate change has been proposed that still serves to illustrate the salient features of the problem. By means of 2 analogues for global climate change, an indirect validation of the model has been performed. First, the spatial analogue has been employed to demonstrate that the spatial patterns in the frequency of extreme temperature events are consistent with the statistical model. Research in hydrology indicates that the spatial patterns in the frequency of extreme precipitation events are consistent as well. Second, the urban heat island has been utilized to indicate that the observed temporal trends in the frequency of extreme temperature events can only be explained by reliance on a model at least as complex as the one introduced here. Of course, it would be helpful to study other instances of human-induced climate modification.

Modifications and extensions of the statistical model could be treated to avoid some limitations of the approach. As already has been shown, changes in the shape of the distribution of climate variables, such as total precipitation, can be permitted by judicious choice of the level at which the climate change is introduced. As only alluded to in the present paper, much research remains to be performed on the related issue of devising appropriate methods for generating scenarios of future climate. Nevertheless, any progress toward producing an acceptable statistical paradigm for climate change would remove one obstacle currently hindering any research efforts that deal with climate change and its potential impacts on society.

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