*The following supplement accompanies the article*

# Abatement potential and shadow price of carbon dioxide emissions from fuel combustion in China's secondary industry

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## Supplement

### Shadow prices of undesirable outputs

The shadow prices of undesirable outputs can be derived from the duality between the DDF and the revenue function (Du et al. 2015). Considering only one desirable output and one undesirable output, $p$ and $q$ represent the price of desirable output and undesirable output respectively. Following Färe et al. (2005), the revenue function of a DMU can be specified as follows:

$$R\left(\boldsymbol{x},p,q\right)=\max_{y,b}\left\{py-qb:\vec{D}\left(\boldsymbol{x},y,b;g_y,-g_b\right)\geq 0,\ \left(y,b\right)\in T\right\} \tag{S1}$$

If $\left(y,b\right)\in T$, then $\left(y+\beta g_y,b-\beta g_b\right)\in T$. Thus, the revenue function can be rewritten as follows:

$$R\left(\boldsymbol{x},p,q\right)\geq p\left(y+\beta g_y\right)-q\left(b-\beta g_b\right) \tag{S2}$$

Given $\beta=\vec{D}\left(\boldsymbol{x},y,b;g_y,-g_b\right)$, Eq (S2) changes to

$$R\left(\boldsymbol{x},p,q\right)\geq\left(py-qb\right)+p\cdot\vec{D}\left(\boldsymbol{x},y,b;g_y,-g_b\right)\cdot g_y+q\cdot\vec{D}\left(\boldsymbol{x},y,b;g_y,-g_b\right)\cdot g_b \tag{S3}$$

The left side of the Eq. (S3) represents the largest possible revenue whereas the right side represents the actual revenue plus the technical efficiency gains. The improvement of technical efficiency includes an increase in desirable output along $g_y$ and a decrease in undesirable output along $g_b$. Eq. (S3) can be rearranged as:

$$\vec{D}\left(\boldsymbol{x},y,b;g_y,-g_b\right)\leq\frac{R\left(\boldsymbol{x},p,q\right)-\left(py-qb\right)}{pg_y+qg_b} \tag{S4}$$

Therefore, the DDF of Eq. (2) in the text can be derived from the revenue function.

$$\vec{D}\left(\boldsymbol{x},y,b;g_y,-g_b\right)=\min_{p,q}\left\{\frac{R\left(\boldsymbol{x},p,q\right)-\left(py-qb\right)}{pg_y+qg_b}\right\} \tag{S5}$$

The following first-order conditions can be obtained by applying the envelope theorem twice to Eq. (S5):

$$\nabla_y\vec{D}\left(\boldsymbol{x},y,b;g_y,-g_b\right)=\frac{-p}{pg_y+qg_b} \tag{S6}$$

$$\nabla_b\vec{D}\left(\boldsymbol{x},y,b;g_y,-g_b\right)=\frac{q}{pg_y+qg_b} \tag{S7}$$

Given the directional vector $\left(g_y,-g_b\right)=\left(1,-1\right)$ and the market price $p$ of desirable output, we can calculate the shadow price of undesirable output.

$$q=-p\left[\frac{\partial\vec{D}\left(\mathbf{x},y,b;1,-1\right)\Big/\partial b}{\partial\vec{D}\left(\mathbf{x},y,b;1,-1\right)\Big/\partial y}\right] \tag{S8}$$

Based on Eq. (4), the shadow price of $CO_2$ emissions for each province per year becomes:

$$q_k^t=-p_k^t\frac{\gamma_1+\gamma_2b_k^t+\sum_{n=1}^{4}\eta_nx_{nk}^t+\mu y_k^t}{\beta_1+\beta_2y_k^t+\sum_{n=1}^{4}\delta_nx_{nk}^t+\mu b_k^t} \tag{S9}$$

which is Eq. (7) in the text.

## References

Du L, Hanley A, Wei C (2015). Marginal Abatement Costs of Carbon Dioxide Emissions in China: A Parametric Analysis. Environ. Resource Econ. 61 191-216.

Färe R, Grosskopf S, Noh D-W, Weber W (2005). Characteristics of a polluting technology: theory and practice. J. Econometrics 126 469-492.