The following supplements accompany the article

Increasing temperature may shift availability of euphausiid prey in the Southern Ocean

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Marine Ecology Progress Series 588: 59–70 (2018)

Supplement 1: Derivation of coefficients for *Thysanoessa macrura* quadratic growth model

In the growth model of Atkinson et al. (2006), growth of *E. superba* is a quadratic function of length and temperature. We use a similar structure for the growth of *T. macrura.* In deriving coefficients for the temperature- and length-dependent *T. macrura* growth model, we make the following assumptions:

1) Maximum possible growth rate is DGR_{max}=0.12 mm day^{-1}. This is the maximum growth rate observed by Driscoll et al. (2015).
2) L_{inf}=40 mm, the asymptotic size for *T. macrura* proposed by Driscoll (2013).
3) The maximum temperature at which growth is possible (ie DGR is positive) is T_{crit}=7 °C, near the maximum temperature in the range of *T. macrura*. Thus, the temperature T_{max} where growth rate is maximized for a given size is 4 °C (model 1) or 3.5°C (model 2).
4) For a given temperature, peak growth occurs at L_{max}=9 mm, or approximately 25% of maximum size. We chose this value because *E. superba* growth is likewise maximized around 25% of maximum size according to Eqn. 5.

Recall that a quadratic equation can be written in the form

$$y = j(x-h)^2 + k \qquad \text{(S1.1)}$$

where (*h,k*) is the vertex. If the intercept (*n,0*) is known, we can solve for *j* as

$$j = \frac{-k}{(p-n)^2} \qquad \text{(S1.2)}$$

and rearrange the quadratic into the form

$$y = jx^2 - 2jhx + jh^2 + k \qquad \text{(S1.3)}$$

To solve for the parameters in Eqn. 6, I set $T=T_{max}$, so that Eqn. 6 becomes

$$DGR(L, T_{\max}) = cL^2 + bL + a' \qquad \text{(S1.4)}$$

where the unknown constant $a' = gT_{\max}^2 + fT_{\max} + a$. This can also be represented in vertex form as

$$DGR(L,T_{\max}) = j\left(L - L_{\max}\right)^2 + DGR_{\max} \qquad \text{(S1.5)}$$

Using the intercept $\left(L_{\inf}, 0\right)$ we see that

$$DGR(L,T_{\max}) = jL^2 - 2jL_{\max}L + jL_{\max}^2 + DGR_{\max} \qquad \text{(S1.6)}$$

where from Eqn. S3 we have $j = -\dfrac{DGR_{\max}}{\left(L_{\inf} - L_{\max}\right)^2}$.

Thus, for the growth equation

$$DGR_m(L,F,T) = a + bL - cL^2 + fT - gT^2 \qquad \text{(S1.7)}$$

we can see that $c = j$ and $b = -2jL_{\max}$.

Setting $L=L_{max}$ we can solve for g and f analogously using the vertex (T_{max}, DGR_{max}) and the intercept (T_{crit}, 0). We find that $g = -\dfrac{DGR_{\max}}{\left(T_{crit} - T_{\max}\right)^2}$ and $f = \dfrac{2DGR_{\max}T_{\max}}{\left(T_{crit} - T_{\max}\right)^2}$. Once b, c, f, and g are known, it is then simple to solve for a, as

$$a = DGR_{\max} - cL_{\max}^2 - bL_{\max} - gT_{\max}^2 - fT_{\max}.$$

Supplement 2: The Metabolic Theory of Ecology

According to the Metabolic Theory of Ecology (MTE), metabolic rates depend on both organism size and temperature (Brown et al. 2004). Because metabolic reactions proceed faster at elevated temperatures, the relationship between temperature and metabolic rate is characterized by the Boltzmann factor $e^{\frac{-E_a}{k_B T}}$, where E_a is the mean activation energy of metabolism, k_B is the Boltzmann constant, and T is absolute temperature. According to the von Bertalanffy growth equation, k is a parameter describing catabolic processes. Thus, according to the MTE k may vary with ocean temperature T_C (in °C) as

$$k = k_0 \exp\left(\frac{-E_a}{k_B\left(273 + T_C\right)}\right) \quad \text{(S2.1)}$$

where k_0 is a constant. This can be rearranged into

$$k = k_0 \exp\left(\frac{-E_a}{k_B} \cdot \frac{1}{273} \cdot \frac{1}{1 + \frac{T_C}{273}}\right) \quad \text{(S2.2)}$$

Using the Taylor approximation $\frac{1}{1+x} \approx 1 - x$ (when x small), this then becomes

$$k \approx k_0 \exp\left(\frac{-E_a}{k_B}\left(1 - \frac{T_C}{273}\right)\right) \quad \text{(S2.3)}$$

and can be rearranged as

$$k \approx k_0 \exp\left(\frac{-E_a}{273 k_B}\right)\left(1 + \left(\frac{E_a}{273^2 k_B} T_C\right)\right) \quad \text{(S2.4)}$$

With these assumptions, k is a linear function of temperature in the same form as Eqn. 8.