Supplement: Model parameter values and equations

Following Ellner & Rees (2006), the population density (ind./m²) at time $t+1$ for a given length $l$ is described by the following equation:

\[ n_{t+1}(l) = G(l|L_0)R_t + \int (S(l', l)G(l'|l))n_t(l')dl' \]  

(S.1)

where: $n_t(l)$ is number of individuals of size $l$ at time $t$; $R_t$ are the annual recruits at time $t$; $L_0$ is the initial length of recruits; $G(l'|l)$ is the growth function, i.e. the probability to growth from size $l'$ to size $l$ in one year; and $S(l'|l)$ is the survival function, accounting for both size-dependent natural mortality and fishing mortality. Note that initial survival for new recruits is already incorporated into the $R_t$ value. Growth, survival, fecundity, and catch functions are defined below.

Growth

We used a probabilistic non-negative Gompertz growth function to model annual green abalone growth (Figure 2a) (Gompertz 1825, Bardos 2005), parameterized by Rossetto et al. (2015). This model allows for variable growth dynamics by creating a distribution of possible growth increments $\Delta l$ for any specific starting length $l$ and allowing maximum length $L_\infty$ to vary dynamically with $l$ (further discussion in Aalto et al. 2019). The mean and variance of $L_\infty$ are determined by $L_n$ and $\sigma_L$, respectively, with associated scaling parameters $\beta$ and $\gamma$. Thus, the probability of an abalone of length $l$ growing by the length increment $\Delta l$ is:
\[ G(\Delta l|l) = \frac{\lambda^p}{\Gamma(p)} (L_{\infty} - l)^{p-1} e^{-\lambda (L_{\infty} - l)} \frac{1}{1-e^{-\lambda}} \left( \frac{\Delta l + l}{l} \right)^{\frac{1}{e^{\lambda} - 1}} \]  

(S.2)

where \( G \) is the Gompertz growth parameter, \( \Gamma() \) is the gamma function, and:

\[ L_{\infty} = \left\{ (l + \Delta l)^{1-e^{-\lambda}} \right\}^{\frac{1}{1-e^{-\lambda}}} \]  

(S.3)

\[ \lambda = \frac{\ln L_n}{\ln (1 + (\beta L_n)^{1+\gamma L_n})^2} \]  

(S.4)

\[ \rho = \left( \frac{L_n}{1+\beta L_n} \right)^{1-\gamma L_n} \]  

(S.5)

**Survival**

The instantaneous mortality rate \( \mu(l) \) for abalone in natural environments is related linearly to body mass \( w(l) \) (Figure 2b; Rossetto et al. 2013), which can be estimated from length (Shepherd 1998):

\[ \ln \mu(l) = \theta + \alpha_w \ln w(l) \]  

(S.6)

\[ w(l) = a_w * l^{b_w} \]  

(S.7)

where \( \theta, a_w, \) and \( b_w \) are scaling parameters. The scaling parameter \( \alpha_w \) is, by default, the same for all lengths, but when testing sensitivity to changes in survival we split it into three sub-parameters: juvenile survival \( \alpha_J \), for \( l < 50 \text{mm} \); immature adult survival \( \alpha_Y \), for \( l \geq 50 \text{mm} \) and \( <L_{\text{mat}} \); and mature adult survival \( \alpha_M \) for \( l \geq L_{\text{mat}} \).
To estimate annual survival for an individual, we integrated instantaneous survival from starting length $l_0$ to ending length $l_1$ and assumed that all harvest occurs at the end of the year:

$$S(l_0, l_1) = \begin{cases} (1 - h) \int_{l_0}^{l_1} e^{-\mu(l)} dl & \text{if } l_1 > L_{\text{minH}} \\ \int_{l_0}^{l_1} e^{-\mu(l)} dl & \text{otherwise} \end{cases} \quad (S.8)$$

where $h$ is the constant annual harvest proportion based on the assumption of constant effort and $L_{\text{minH}}$ is the minimum harvest size (see Catch below).

**Fecundity**

The number of eggs produced at time $t$ is calculated as follows from eggs per unit mass $e_w$, and the total mature biomass (Tutschulte 1976, Rossetto et al. 2013), multiplied by $\frac{1}{2}$ to track only females and modified by the Allee effect $Allee_t$:

$$E_t = 0.5 * Allee_t * e_w \int p_{\text{mature}}(l) w(l) n_t(l) dl \quad (S.9)$$

with egg production increasing exponentially with length (Figure 2c). The proportion of reproductive individuals for a specific length $l$ is:

$$p_{\text{mature}}(l) = 0.5 * \frac{1}{1 + e^{-\frac{l-L_{\text{mat}}}{a_{\text{mat}}}}} \quad (S.10)$$

where $a_{\text{mat}}$ is a scaling parameter and $L_{\text{mat}}$ is the length at which half of individuals are mature (Rossetto et al. 2013).
In order to assess possible OA effects on the ability of a population to recover from a mass-mortality event, we included an Allee effect in the fecundity function. Following Button (2008), the Allee effect at time $t$ $Allee_t$ (Figure 2d) is modeled as a relative breeding success multiplier bounded between 0 and 1 and determined by mean aggregation size $Agg_t$:

$$Allee_t = P_{\text{mixed}}(Agg_t).$$  \hfill (S.11)

$Agg_t$ is calculated using the total density of mature individuals $M_t$:

$$M_t = \int p_{\text{mature}}(l)n_t(l)dl$$  \hfill (S.12)

to estimate mean aggregation size (Button 2008):

$$Agg_t = a_{agg} \times M_t + b_{agg}$$  \hfill (S.13)

and used to determine the probability of mixed gender aggregations:

$$P_{\text{mixed}}(Agg_t) = 1 - 0.5^{Agg_t^{-1}}.$$  \hfill (S.14)

The aggregation regression parameters $a_{agg}$ and $b_{agg}$ were derived from Button (2008) by setting the 80% breeding success threshold for the Allee effect to 0.2 mature adults/m$^2$. This value is similar to that seen in prior work (0.15-0.2, Shepherd & Brown 1993, Shepherd & Partington).
1995; 0.3, Babcock & Keesing 1999) and consistent with recruitment failure seen in the field (failure at 0.003 vs. success at 0.85, Rogers-Bennett et al. 2004).

Recruitment

The number of larvae settling at time $t$ was calculated as follows:

$$S_{z,t} = r(t)\sigma_E E_{f,t}$$  \hspace{1cm} (S.16)

where $\sigma_E$ represents the survival from eggs to settlers, and $r(t)$ is a random variable with mean =1 and log-normal distribution that represents relative annual fluctuations in settler survival rate due to year-to-year environmental variability (Szuwalski et al. 2015).

Following Micheli et al. (2008) and Rossetto et al. (2013), we modeled successful recruitment $R_t$ as a Ricker function (Ricker 1954) of the form:

$$R_t = \alpha S_t \exp\left(\frac{-S_t}{K}\right)$$ \hspace{1cm} (S.17)

with density-dependence occurring only at the settlement stage and $r(t)$ in Eq.16 accounting for year-to-year environmental variability. We set carrying capacity $K$ to produce an unfished density of ~0.2 ind./m$^2$, the assumed Allee threshold, based on the work of McShane (1991) and Daume et al. (2004) and consistent with field observations (Rogers-Bennett et al. 2004, Micheli et al. 2008).
Catch

Catch is calculated in metric tons as:

\[ C_t = 10^{-6} A \int_{l_{\text{minH}}}^{l_{\text{max}}} hw(l)n_{z,e}(l) \, dl \]  

(S.18)

where \( A \) is the area of the fishery and \( h \) is the proportion of commercial size abalone that is harvested in year \( t \), representing a constant level of harvest. We set harvest proportion \( h \) to 2/3 of \( F_{\text{MSY}} \) as a precautionary compromise.

Table S1: Model parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Source</th>
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<tbody>
<tr>
<td>Population dynamics</td>
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<tr>
<td>Length at recruitment</td>
<td>( L_0 )</td>
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<td>mm</td>
<td>a</td>
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<td>Length for 50% maturity</td>
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<td>Minimum harvest length</td>
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<td>a</td>
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<tr>
<td>Annual harvest proportion</td>
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<td>Weight scaling value</td>
<td>( a_w )</td>
<td>( 2.24 \times 10^{-5} )</td>
<td>g mm(^{-bw} )</td>
<td>c</td>
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<td>Weight exponent</td>
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<td>No-shell wt. proportion</td>
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<td>( G )</td>
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<td>Gomp. max length mean</td>
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<td>Gomp. max length var.</td>
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<td>Gomp. var. scaling</td>
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<td>Mortality scaling</td>
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<td>Fecundity</td>
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<td>Slope for aggregation est.</td>
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<td>Intercept for agg. est.</td>
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<tr>
<td>Survival of recruits</td>
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<td>Carrying capacity for recruits</td>
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<td>indiv.</td>
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<td>Recruit. log-normal sd</td>
<td>( r_{sd} )</td>
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</tr>
</tbody>
</table>
a. From Rossetto et al. 2015.
b. Harvest proportion was set to 2/3 of \( F_{\text{MSY}} \).
c. From Shepherd et al. 1998
d. Parameters from Bardos 2005 model estimated by fitting observed growth data as described in Rossetto et al. 2015.
e. The mortality parameters are for \textit{Haliotis fulgens} from Rossetto et al. 2012. Similar to non-size-dependent values seen in other abalone literature \( (M=0.38, \text{Shepherd 1990}; M=0.77, \text{Button } \& \text{Rogers-Bennett 2011}) \). The default value for each age class (juvenile, young adult, mature adult) is the same.
f. From Tutschulte 1976.
g. The aggregation regression is from Button 2008 with slight alterations. \( \) We changed \( a_{\text{agg}} \) from 11.3 to 11.6, a value within the given 95% uncertainty range, to set 80% breeding success to 0.2 \text{ind./m}^2. \) We set the intercept \( b_{\text{agg}}=1 \) instead of 1.959 so that aggregation size =1 when density approaches 0.
h. The original egg survival value from Rossetto et al. 2013 =3.09x10^{-3}. \) We adjusted this survival value by 1.846 to remove the implicit Allee effect estimated from the observed densities of 0.015 \text{ind./m}^2.
i. The original estimated value of \( K \) from McShane 1991 and Duame et al. 2004 was 10^7. \) We adjusted this slightly to produce a mean unfished density of \(~0.2 \text{ind./m}^2. \) 
j. Following the observations of Shepherd 1990, we set these values to produce an \(~15x \) ratio between highest and lowest recruitment years after trimming outliers.

\textbf{Literature cited}


Button CA (2008) The influence of density-dependent aggregation characteristics on the population biology of benthic broadcast-spawning gastropods: Pink abalone (Haliotis corrugata), red abalone (Haliotis rufescens), and wavy turban snails (Megastraea undosa). University of California, San Diego


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