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Drought analysis based on a cluster Poisson model: distribution of the most severe drought

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ABSTRACT: This work aims to characterize the largest drought event to occur in a given period of time. A Poisson cluster process is used to model drought occurrence and a vector of 3 random variables (duration, deficit and maximum intensity) to describe their severity. Some results on the distribution of the maximum in a random size sample are developed in order to describe the largest drought events.

KEY WORDS: Drought analysis · Poisson cluster process · Maximum in a random size sample · Threshold methods · Extreme values

1. INTRODUCTION

An in-depth study of drought is of interest since it is a serious problem that recurrently affects many regions, in particular most southern and eastern areas of Spain. The description of drought characteristics by probability distributions provides measures such as inter-drought recurrence time, expected duration or mean deficit, which are helpful in water-resource management.

In previous works on drought analysis, different approaches have been applied: run theory (Sen 1976, Moyé et al. 1988) and renewal processes (Kendall & Dracup 1992). We opted for an approach based on the excess over threshold (EOT) methodology (Zelenhasic & Salvai 1987, Madsen & Rosbjerg 1995) and extreme value theory (EVT), which allows us not only to describe drought but also to characterize the largest drought event to occur in a given period of time.

Below-normal precipitation over a period of time is usually the first sign of a drought. Rainfall shortage may not only cause meteorological drought but also, depending on its duration and intensity, have other consequences, giving rise to hydrological or agricultural drought. Herein, only meteorological drought is analyzed, but the methodology employed can be applied to the study of other aspects and definitions of the phenomenon. The analysis of the monthly rainfall series of Huesca, a location situated in the northeast of Spain, is shown as an example. The length of this time series is 136 yr.

2. MODEL OF THE DROUGHT PROCESS

2.1. Drought definition

In general, meteorological drought can be defined as a deficiency of precipitation over an extended period of time resulting in a water shortage. In practice, however, the drought definition must reflect differences caused by climate, regional characteristics and requirements; so, no single definition works in all circumstances. Operational definitions that allow us to identify the beginning, end and degree of severity of the drought are based on the EOT approach, in which a stochastic process, $s(t)$, related to precipitation or some other variable describing the hydric state of the system, is compared to a threshold, $U1(t)$, which represents a critical level for the process. A drought will occur when $s(t)$ is below $U1(t)$, as shown in Fig. 1.

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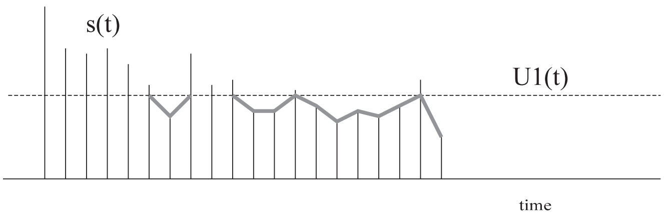

Fig. 1. Dry event definition. $s(t)$: a stochastic process related to precipitation or some other variable describing the hydric state of the system; $U1(t)$: a critical level for the process

Since drought is a phenomenon that requires a period of time to pass before it is noticed, we used, as signal $s(t)$, a monthly moving series where each observation is the accumulated rainfall in the p previous months. The use of different values of p allows us to characterize short- and long-term droughts. In this work p equals 12 mo; thus, the series will reflect water-resource deficiencies in processes based on long-term precipitation, such as reservoir levels. Drought effects in other fields, dry farming for example, should be based on series with a shorter accumulation period. Whichever accumulation period we use, the monthly update is strongly recommended, since it allows a frequent drought-intensity evaluation.

Our approach for defining the drought threshold is based on the decile method proposed by Gibbs & Maher (1967). The threshold is defined as a constant value equal to a percentile of the rainfall series. According to the Gibbs & Maher classification, values below the tenth percentile, denoted by $p10$, correspond to an extremely dry period. Obviously, a constant threshold can only be used when there is no seasonal component in the series.

We are not only interested in characterizing drought occurrence, but also its severity; to describe this, we use 3 variables: duration or length, L; accumulated deficit to the threshold, D; and maximum intensity during the drought, MI.

To illustrate the previous drought definition and to show what the Huesca series looks like, the differences from the rainfall series $s(t)$ to the $p10$ threshold is represented in Fig. 2. The rainfall amount is measured in decilitres (dl), and a monthly time scale is used on the x-axis.

2.2. Extreme value theory

Well-known EVT results (e.g. Davison & Smith 1990, Embrechts et al. 1997), assert the following:

- The occurrence of excesses in independent or short-term-dependent stationary processes converges to a homogeneous Poisson process (PP) when high-enough thresholds are used.
- The distribution of the excess amount converges to a generalized Pareto (GP) distribution.

According to these results, the occurrence of dry periods could be properly modelled by a PP if we define them by using a sufficiently extreme threshold. Since the PP is a point process, an occurrence point must be assigned to each dry period; in the checking analysis similar results were obtained using the initial, central and maximum intensity points in the period.

The validity of the model was checked using 6 long Spanish rainfall series: Burgos, Daroca, Huesca, Madrid, Murcia and San Fernando. Just as an example, we present some results from the Huesca series. More details on the analysis for Huesca and the other series can be found in Cebrián (1999).

2.3. Threshold selection

The first step in applying the model is to select a proper threshold, and this is not a trivial task since the only available tools are graphical methods. In the interpretation of the graphs we have to bear in mind that we are dealing with deficits to a threshold instead of excesses, the type of extreme events usually analyzed by the EOT approach. Consequently, herein, an extreme threshold is not a high but a low one, and the observations we are interested in are $u - X \mid X \leq u$ and not $X - u \mid X \geq u$; however, we shall retain the traditional EOT terms, such as excess, exceedances, etc., instead of terms related to shortage.

The graphical tools are based on properties of the PP or the GP distribution. In Fig. 3a, the mean-variance ratio of the number of events in a period of time, T, which in a PP must be 1, is represented as a function of the threshold. The record of the series was divided into 18 periods of length $T = 90$ mo to calculate the sample ratios represented in the plot.

Fig. 3b is the mean excess plot, where, in this case, an empirical estimation of $E[u - X \mid X < u]$ is represented as a function of the threshold; when the exceedances follow a GP distribution, the mean excess is a linear function of the threshold. Given the linearity observed from values slightly below $u = 4000$ dl, the extreme behaviour of the exceedances can be assumed using this value or any more extreme (lower) threshold. According to these results, a threshold equal to the tenth percentile (3700 dl) suggested by Gibbs & Maher (1967) was selected; the corresponding dry-period series is denoted by $dp10$.

To confirm the validity of the threshold, we check the distribution of the distance between 2 consecutive events, which must be exponential in a PP. In Fig. 4 the exponential qq-plot of inter-event distances in the $dp10$ series is shown. In spite of the 2 largest observa-

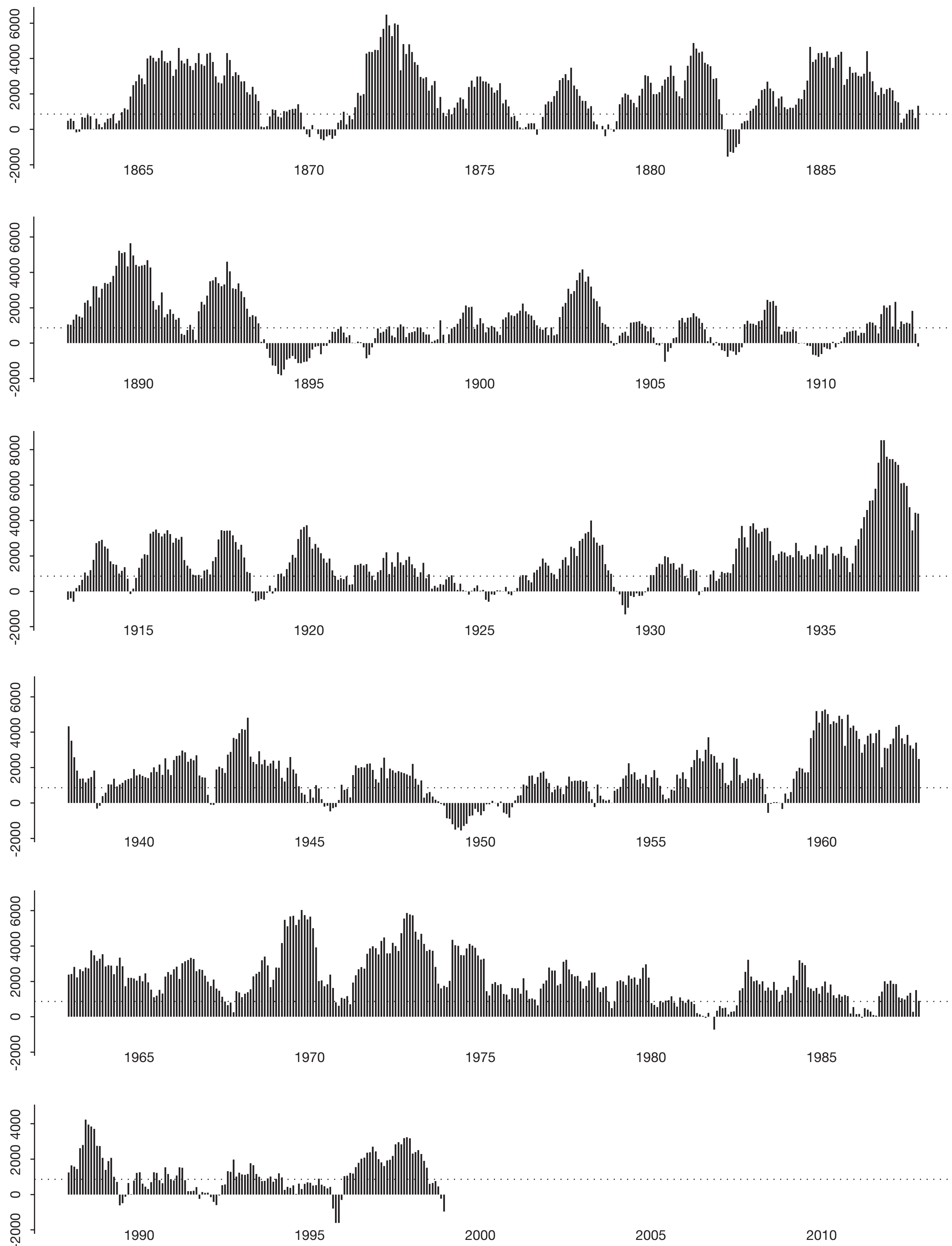

Fig. 2. Series of differences between $s(t)$ and the threshold $U1$ = 3740 dl with a monthly update (Huesca). Dotted line: $U2$, equal to the thirtieth percentile (4600 dl)

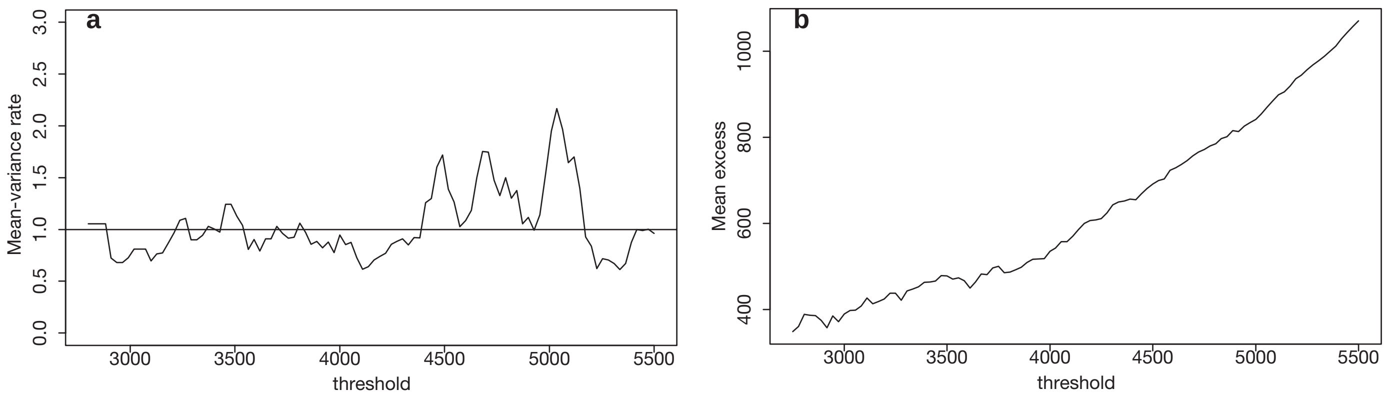

Fig. 3. (a) Mean-variance ratio and (b) mean excess plots for the Huesca series

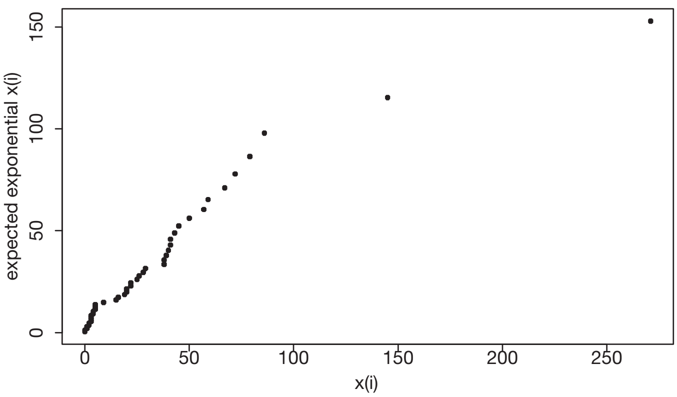

Fig. 4. Exponential *qq*-plot of the inter-drought distances for the Huesca *dp*10 series

tions in the sample, the exponential character cannot be rejected according to the Kolmogorov-Smirnov goodness-of-fit test (p-value = 0.126).

Once the dry periods are defined, the parameter λ of the PP can be easily estimated by maximum likelihood from the sample of the inter-event distances.

The checking of the PP as an occurrence model was satisfactory for the 6 *dp*10 series analyzed (see Cebrián 1999); only certain homogeneity problems were detected in San Fernando, but these were not statistically significant at the 5 % level.

2.4. Characterization of dry period severity. Clustering of the events

A characterization of drought severity is needed to complete the model, so a random vector formed by 3 variables representing the event duration (L), the accumulated deficit (D) and the maximum intensity observed during the dry period (MI) was defined. The analysis of these variables revealed that the dry events could not be considered independent, since some kind of dependence exists between the severity of closed events. More precisely, significant auto-correlation was found in the deficit and the maximum intensity series; the Kendall τ coefficient for the first-order auto-correlation of the 3 severity series and the corresponding p-values from testing the hypothesis H_0: $\tau = 0$ can be seen in Table 1.

This auto-correlation is caused by the signal we used and the characteristics of the drought process; in fact, during a lengthy drought period it can be observed that the signal slightly exceeds the threshold for a short period of time, dividing a large drought into a number of minor dry spells. Such a cluster of dry spells should be treated as 1 drought as long as its impact is not eliminated by a non-dry period. Such a clustering of dry spells can be observed in Fig. 2.

2.5. Poisson cluster process

One model that allows us to represent this structure is the Poisson cluster process (PClP), in which clusters representing droughts occur according to a PP and are formed by a random number of points, corresponding to dry spells which form a subsidiary process. The number of events per cluster must be independent and identically distributed. The use of a PClP for modelling

Table 1. Kendall's τ and p-values of the auto-correlation test for the Huesca *dp*10 series

	Duration	Deficit	Max. intensity
Kendall's τ	–0.107	–0.256	–0.281
p-value	0.263	0.015	0.007

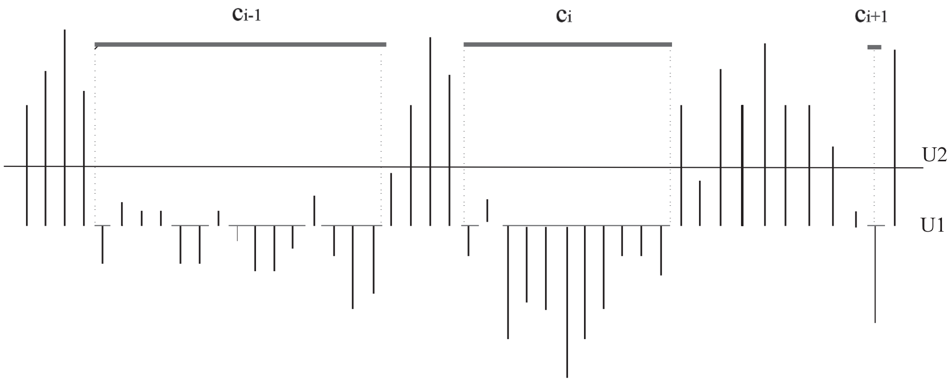

Fig. 5. Cluster composition diagram

drought occurrences using extreme enough thresholds is asymptotically justified in Cebrián (1999).

The main drawback of this model is the difficulty in determining the cluster composition. Davison & Smith (1990) suggested the use of parametric models such as the Neyman-Scott and Bartlett-Lewis models; but they conclude that, in general, such models do not improve the results obtained using empirical rules. Moreover, since we are interested in characterizing the whole cluster (the drought) and not the internal distribution of their points (the dry spells), we did not consider these types of models. After trying out several criteria, we used an empirical rule based on those of Madsen & Rosbjerg (1998) and Rasmussen et al. (1994), but taking into account more information about the separating non-dry periods. Two dry events are considered to belong to the same cluster if

- the time between their occurrence is less or equal to 6 mo;
- no intensity value during that time reaches a value $U2$ equal to the thirtieth percentile, considered as a normal rainfall value according to the Gibbs & Maher (1967) classification.

The value $U2$, which for the Huesca series equals 4600 dl, has been represented in Fig. 2 as the constant line $y = 860$; this value is the difference to the threshold $U1 = 3740$ dl. This helps us to observe the clusters formed in the Huesca series. A more detailed example of clusters defined in this way is shown in Fig. 5. The series of droughts obtained by applying this definition will be denoted *drp*10.

The PClP was applied to the 6 former series and its fitting was satisfactory with the tenth percentile as the threshold in Burgos, Daroca and Huesca; a more extreme threshold, about the sixth percentile, is required in Madrid and Murcia in order to make the model work. In San Fernando, a seasonal behaviour was detected in the occurrence process of the droughts.

The exponential *qq*-plot for recurrence times between droughts in the Huesca series is shown in Fig. 6. Again, 2 observations show a deviation from the distribution, but the Kolmogorov-Smirnov test for exponentiality was not rejected (p-value = 0.144).

2.5.1. Drought severity

As in the dry-event process, the drought-occurrence model must be completed with a characterization of the drought severity. Again, a vector formed by the 3 variables L, D and MI of the cluster is considered. However, a new definition of deficit is needed, since now a drought can be formed by more than 1 dry event and the rainfall amount during the separating non-dry periods must be taken into account. To solve this question we finally opted for defining the deficit with respect to another threshold, $U2$ (the normal rainfall value $p30$), instead of using $U1$ (the threshold used to define dry spells). This definition represents the drought severity better than the one suggested by Zelenhasic & Salvai (1987), and it cannot assume negative values, such as the one by Madsen & Rosbjerg (1998). The definitions of the 3 variables are illustrated in Fig. 7.

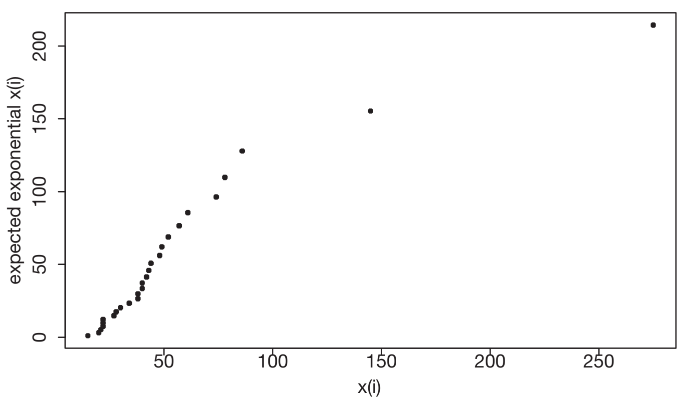

Fig. 6. Exponential *qq*-plot of the inter-drought distances for the Huesca *drp*10 series

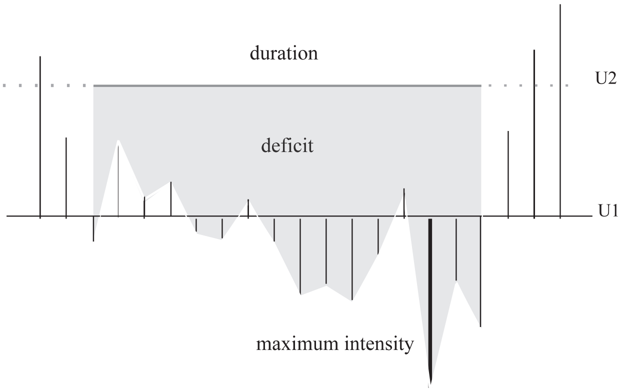

Fig. 7. Severity variables associated with a drought (cluster)

Table 2. Kendall's τ and p-values of the auto-correlation test for Huesca *drp*10 series

	Duration	Deficit	Max. intensity
Kendall's τ	−0.041	−0.053	−0.074
p-value	0.656	0.613	0.479

The 3 severity series *L*, *D* and *MI* defined for clusters are no longer auto-correlated (see Table 2) nor is any other reason to reject the independence hypothesis found in any case. Concerning homogeneity, the results are satisfactory except in Madrid and San Fernando, where seasonal behaviour is detected. No time trend is observed in any of the series; however, using series $s(t)$ with a shorter accumulation period $p = 3$, Huesca and some other series showed an increase in drought severity from approximately 1970 onwards (see Abaurrea & Cebrián 2001).

Under these conditions, marginal distributions for each variable can be fitted using maximum-likelihood methods. Several positive distributions—exponential, Weibull, Gamma, log-normal, log-logistic and GP distributions—were considered for modelling *D* and *MI*. For *L*, the same continuous distributions (but shifted, since drought length is always ≥ 1) were tested, and discrete distributions such as geometric and Poisson distributions were also considered. Graphical tools, and likelihood tests when possible, were used to select the best distribution and to check the goodness of fit.

For the Huesca series, we found that the duration requires a no-memory distribution such as the shifted exponential distribution; the best fit for the deficit is provided by the log-normal distribution and, for the maximum intensity, by the GP distribution, as expected from EVT results. Details are in Cebrián (1999).

3. DISTRIBUTION OF THE MOST SEVERE DROUGHT TO OCCUR IN A GIVEN PERIOD OF TIME

An interesting issue concerning drought analysis is the characterization of the most severe drought to occur in a given period of time. This information can be useful in water-resource planning in order to help estimate, for example, the required capacity of a reservoir.

In the previous section, the following conclusions were stated:

- Droughts, defined as clusters of dry periods, occur according to a PP with λ intensity, $PP[\lambda]$ in short. This means that the number of droughts in a time period of length n will be a Poisson variable, N, with mean $\lambda_n = \lambda n$.
- Drought severity can be described using $(L_i, D_i, MI_i)_{i=1\ldots N}$, a series of iid random vectors.

The distribution of the maximum *L*, maximum *D* and *MI* in a period of length *n* corresponds to the distribution of the maximum in a series of independent identically distributed (iid) random vectors $\boldsymbol{P}[\lambda_n]$.

3.1. Some previous definitions

3.1.1. Generalized extreme value (GEV) distribution

The GEV distribution, one of the most important distributions in EVT, arises as the limit distribution of the maximum in an iid sample. More precisely, the Fisher-Tippett theorem (Fisher & Tippett 1928) states that the GEV distribution is the only non-degenerate limit distribution for appropriately normalized sample maxima. The cumulative distribution function (cdf) *H* corresponding to the standard GEV distribution is:

$$H(x;\gamma) = \exp\left[-(1+\gamma x)^{-1/\gamma}\right]$$

defined when $1 + \gamma x > 0$.

Depending on the value of the shape parameter, γ, 3 distributions are obtained: Gumbel for $\gamma = 0$, Fréchet for $\gamma > 0$ and Weibull for $\gamma < 0$. The GEV distribution can be easily generalized by including location and/or scale parameters, μ and σ.

3.1.2. Maximum domain of attraction (MDA)

The MDA of a distribution *H*, *MDA*(*H*), is defined as the set of all distributions such that its maximum converges to *H*,

$$\lim_{n\to\infty} P\left(\frac{M_n - b_n}{a_n} < x\right) = H[x]$$

According to the Fisher-Tippett theorem, only the MDA of the GEV distribution can be defined. A thorough review of these and other results of EVT can be found in Embrechts et al. (1997).

3.2. Distribution of M_N in samples of random size $N \sim P[\lambda]$

In an iid sample of size N, the distribution function of the maximum can be expressed in terms of the sample cdf F as $F_{M_N}(x) = F(x)^N$. If N is a random variable and, in particular, if it follows a Poisson distribution with the λ parameter, we obtain the following expression:

$$F_{M_N}(x) = \sum_{i=0}^{\infty} P(N=i)F^i(x) = \exp\{-\lambda[1-F(x)]\} \quad (1)$$

In some cases Eq. (1) provides a simple cdf so that an exact distribution of the maximum can be used, but usually the resulting distributions from Eq. (1) are too complicated and it is better to use the asymptotic results suggested at the end of this section.

3.2.1. Exact cdf of M_N

In the case of the exponential and GP distributions the exact distribution of the maximum can be obtained (see Cebrián 1999).

- The distribution of the maximum, M_N, of a sample of size $P(\lambda)$ and distribution $\exp(\alpha)$ is *Gumbel* $(1/\alpha, \ln(\lambda)/\alpha)$.
- The distribution of the maximum M_N of a sample of size $P(\lambda)$ and distribution $GP(\gamma,\sigma)$ is *GEV* $\left(\gamma, \sigma\lambda^{\gamma}, \frac{\sigma\lambda^{\gamma}-\sigma}{\gamma}\right)$ with the same γ parameter

3.2.2. Asymptotic results for the distribution of M_N

By applying the theorem of Galambos (1978) concerning the limit distribution of the maximum in a random size sample, the following result has been proved (see Cebrián 1999).

Theorem: In a sample with distribution $F \in MDA(Gumbel)$ and size N, where N is a $P(\lambda_n)$-distributed variable whose parameter depends on n, the asymptotic distribution of the normalized maximum, $(M_N - b_n)/a_n$, is *Gumbel* $(1, \ln(\lambda_n))$.

In the drought analysis we are performing, n will be the size of the time series in which the random number of events N is observed.

In particular, from the previous theorem we can obtain the distribution of M_N in Gamma, Weibull, or log-normal samples, since all of them belong to *MDA*(*Gumbel*). These distributions, together with the previously mentioned exponential and GP distributions, are the most frequently used in environmental problems. More details can be found in Cebrián (1999).

This theorem is seldom useful in practical applications due to its asymptotic character and to the fact that large-enough samples are not usually available. In the next subsection, a further approximation providing better results in practical applications is suggested.

3.3. Penultimate approximation

3.3.1. Samples of non-random size n

For non-random size samples, Gomes (1984) and Castillo (1988) state that if $F \in MDA(GEV)$,

- the convergence of the maximum to the limit distribution is very slow;
- a GEV distribution with cdf $H(\gamma_n, s_n, \mu_n)$, with its 3 parameters depending on n, called the *penultimate approximation*, provides a better approximation than the limit GEV distribution with a constant shape parameter, $H(\gamma, \sigma_n, \mu_n)$.

3.3.2. Samples of Poisson size $N \sim P(\lambda_n)$

Given the practical interest of the penultimate approximation, we have developed a similar result for random size samples, in particular for samples with size N following a $P(\lambda_n)$ distribution.

Theorem: Given a sample with cdf $F \in MDA(GEV)$, if the penultimate approximation of M_n (the maximum in a sample of non-random size n) is a GEV distribution, $H(\gamma_n, \sigma_n, \mu_n)$, the cdf of the penultimate approximation of M_N [the maximum in a sample of random size $N \sim P(\lambda_n)$] will be $H^{\lambda n}(\gamma_n, \sigma_n, \mu_n)$, the λ_n power of the cdf $H(\gamma_n, \sigma_n, \mu_n)$.

The following property can also be proved:

Property: If F is the cdf of a $GEV(\gamma,\sigma,\mu)$ distribution, F^a for any $a > 0$ is the cdf of a distribution $GEV[\gamma, \sigma a^{\lambda}, \mu + \sigma(a^{\lambda} - 1)/\gamma]$. Since the shape parameter, γ, in both distributions is the same, F and F^a are GEV distributions of the same type.

Bearing in mind the former results, we obtain the following corollary:

Corollary: In samples with cdf $F \in MDA(Gumbel)$, in particular for Weibull, Gamma or log-normal distributions, the penultimate approximation of M_N is a GEV distribution with $H(\gamma'_n, \sigma'_n, \mu'_n)$, whose 3 parameters (all depending on n) can be estimated using the percentile method. In this method the parameters are calculated by solving an equation system obtained by making the expressions of 3 percentiles of a GEV distribution equal to those obtained from the exact F_{M_N} distribution in Eq. (1).

Proofs and further details for these results can be found in Cebrián (1999).

3.4. Results

As an example we show the results concerning the fitting of the distribution of the most severe drought event expected in a given period of time in Huesca. Validation of long-term predictions with independent data is not possible, since no available series is long enough. Instead, we have made a comparison between predictions of the maximum in n years based on the model and the values observed in the record for n = 50 yr and n = 100 yr, obtained in the following way: The maximum values of L, D and MI observed during the following n years are determined for every observation of the series with enough future data. As there are 1633 monthly observations in the Huesca series, 1034 maximum observations for n = 50 yr and 434 for n = 100 yr are available from the record. Obviously, these observations are not independent, but they allow us at least to perform a certain checking of the model. The median and mean values of the samples are calculated and compared with their parametric counterparts.

We found that the drought duration fitted an exponential distribution, so, according to the result from Section 3.2.1, the distribution of the maximum will be a Gumbel distribution with the parameter values depending on the period of time. The estimated parameters corresponding to periods of 50, 100, 200 and 500 yr are shown in Table 3 (top), together with the mean and median of the distributions and the corresponding empirical values when they are available. The drought deficit fitted a log-normal distribution. Applying the penultimate approximation, its maximum value will follow a GEV distribution whose parameters are shown in Table 3 (center). Finally, the maximum intensity was fitted by a GP distribution; thus, as stated in Section 3.2.1, the maximum will follow a GEV distribution whose parameters are shown in Table 3 (bottom).

Table 3. Fitted parameters (Gumbel distribution for maximum duration and GEV for maximum deficit and maximum intensity) and some observed statistics in the Huesca series

T	50 yr	100 yr	200 yr	500 yr
Occurrence (Poisson process)				
λ_P	11.4	22.8	45.6	114.0
Maximum duration (mo)				
μ	17.55	22.27	26.99	33.22
σ	6.80	6.80	6.80	6.80
$E(M_N)$	21.48	26.20	30.91	37.15
Median	15.1	19.8	24.5	30.7
Observed mean	23.9	26.0	–	–
Observed median	26	26	–	–
Maximum deficit (l)				
γ	0.330	0.308	0.290	0.270
μ	21978.0	32304.1	45219.7	66988.6
σ	13191.2	16643.9	20671.5	26988.9
$E(M_N)$	3589.7	4910.9	6535.0	9227.3
Median	2711.7	3876.2	5321.3	7738.6
Observed mean	3683.3	3829.0	–	–
Observed median	3829.0	3829.0	–	–
Maximum intensity (l)				
γ	−0.50	−0.50	−0.50	−0.50
μ	1436.93	1614.03	1739.27	1850.39
σ	302.34	213.78	151.16	95.61
$E(M_N)$	150.6	166.3	177.4	187.2
Median	153.8	168.6	179.0	188.2
Observed mean	163.1	177.9	–	–
Observed median	155.0	181.2	–	–

4. CONCLUSIONS

A deeper knowledge of the characteristics of the drought process can help to decide which measures should be taken in order to prevent the problem. In particular, the characterization of the severity distribution of the maximum drought expected in a given period of time is an important issue in water management, since it can provide useful information for the design of the required reservoir facilities in a region and a firm basis for decisions concerning the most extreme risk. Mean values and high percentiles, representing critical drought values, can be obtained from the probability distribution of the most severe drought.

We found that one of the problems in drought analysis is the lack of rainfall records long enough for the analysis of such an extreme event. In these circumstances, the suggested approach based on extreme value theory provides an optimum frame. The use of the Poisson cluster model completed by the vector of variables (duration, deficit, maximum intensity) for modelling the drought occurrence and severity was successful. Its validity, suggested by asymptotic results, was confirmed through analysis.

The model is useful not only for describing general drought properties but also as a basis for predicting the most severe drought characteristics observed in a given period of time. The probability distribution of these characteristics, obtained from both the penultimate approximation or the exact distribution of the maximum in Poisson size samples, provide satisfactory predictions according to the observed record. The methodology is widely applicable with common distributions in environmental sciences (e.g. Gamma, log-normal, Weibull).

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