

## **Multifractal analysis of meteorological time series to assess climate impacts**

**Piotr Baranowski\*, Jaromir Krzyszczyk, Cezary Slawinski, Holger Hoffmann, Jerzy Kozyra, Anna Nieróbca, Krzysztof Siwek, Andrzej Gluza**

\*Corresponding author: pbaranow@ipan.lublin.pl

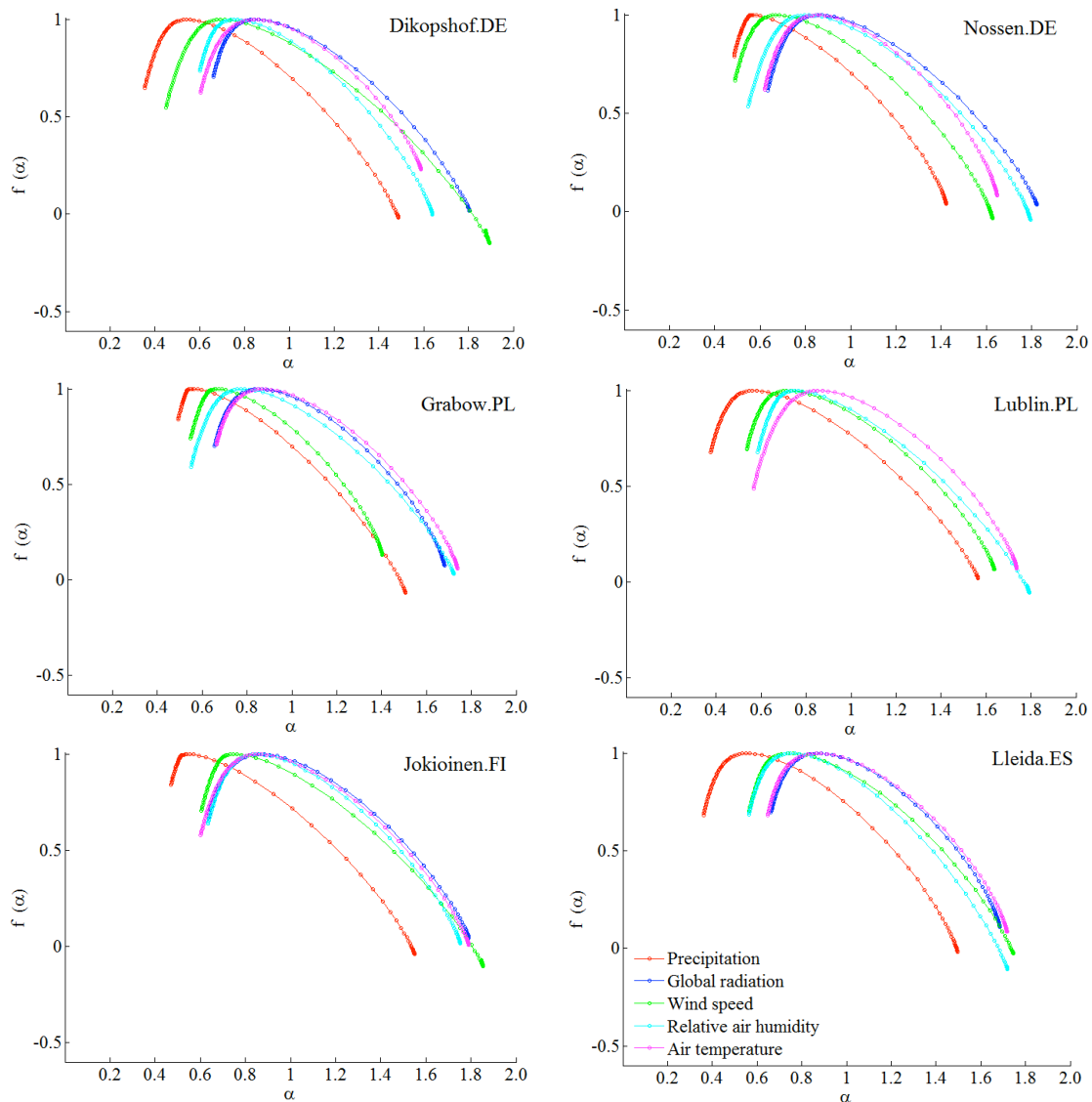
*Climate Research 65: 39–52 (2015)*

---

### **Supplement.**

The use of FT (Fourier Transformed) and AAFT (Amplitude Adjusted Fourier Transformed) surrogate data to distinguish the source of multifractality in meteorological time series

The amplitude adjusted Fourier transform (AAFT) surrogate data were frequently used in literature to distinguish the source of multifractality (Theiler et al. 1992, Schreiber & Schmitz 1996, Schreiber & Schmitz 2000, Nagarajan 2007, Min et al. 2013, Hall 2014, Mali 2014), but also other surrogates can be applied. For example, Fourier transform (FT) surrogates without the amplitude adjustment procedure destroys nonlinear correlations in the analysed signal, leaving only the linear correlations. Therefore, the application of FT randomization should be perfect to distinguish whether the nonlinear or linear long-range correlations are dominant within long-range correlations. If the nonlinear correlations of variability prevail, the FT spectrum of a multifractal time series is very narrow (similar to monofractal spectrum). If the linear correlations of variability prevail, the FT spectrum is very rich (high width) and we can say that the process is strongly nonlinear. Therefore, to compare AAFT and FT procedure, the FT randomization was performed for studied time series. The FT spectra are presented in Fig. S1.

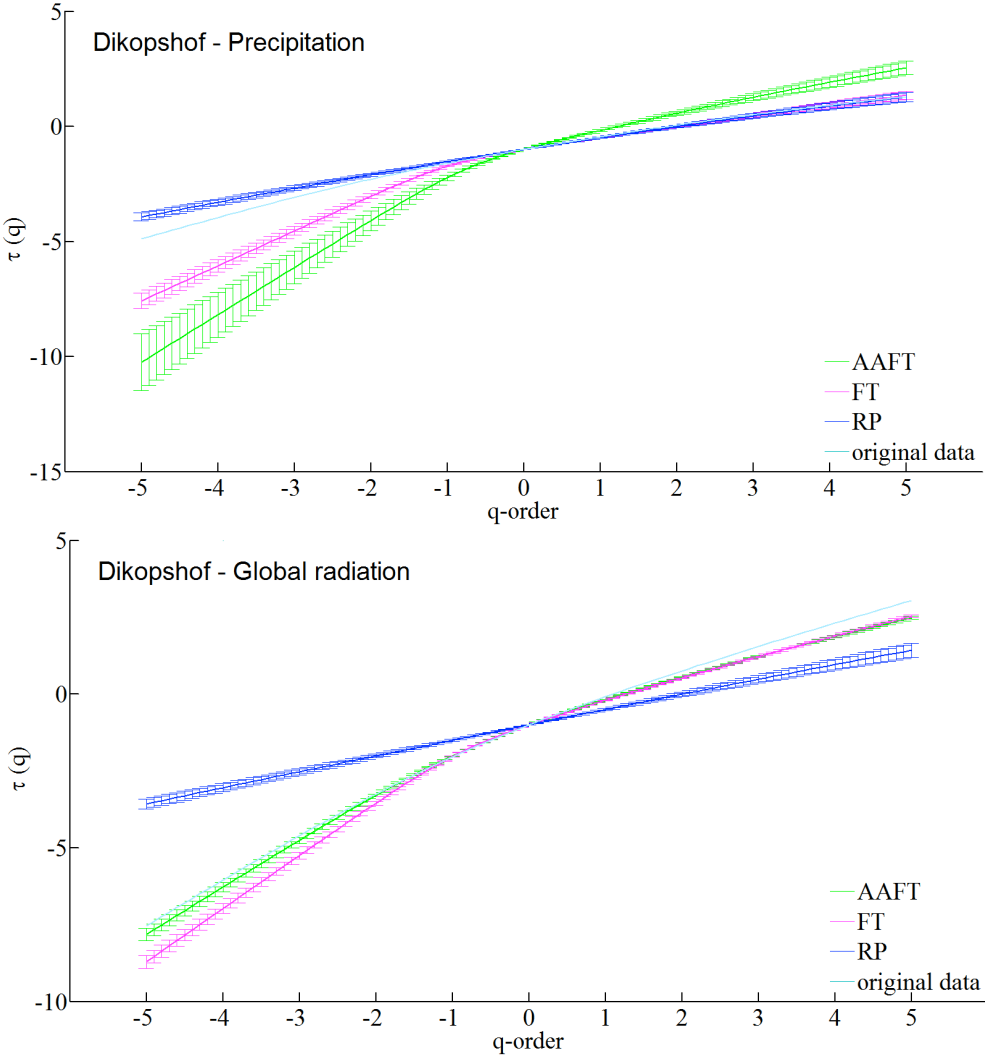


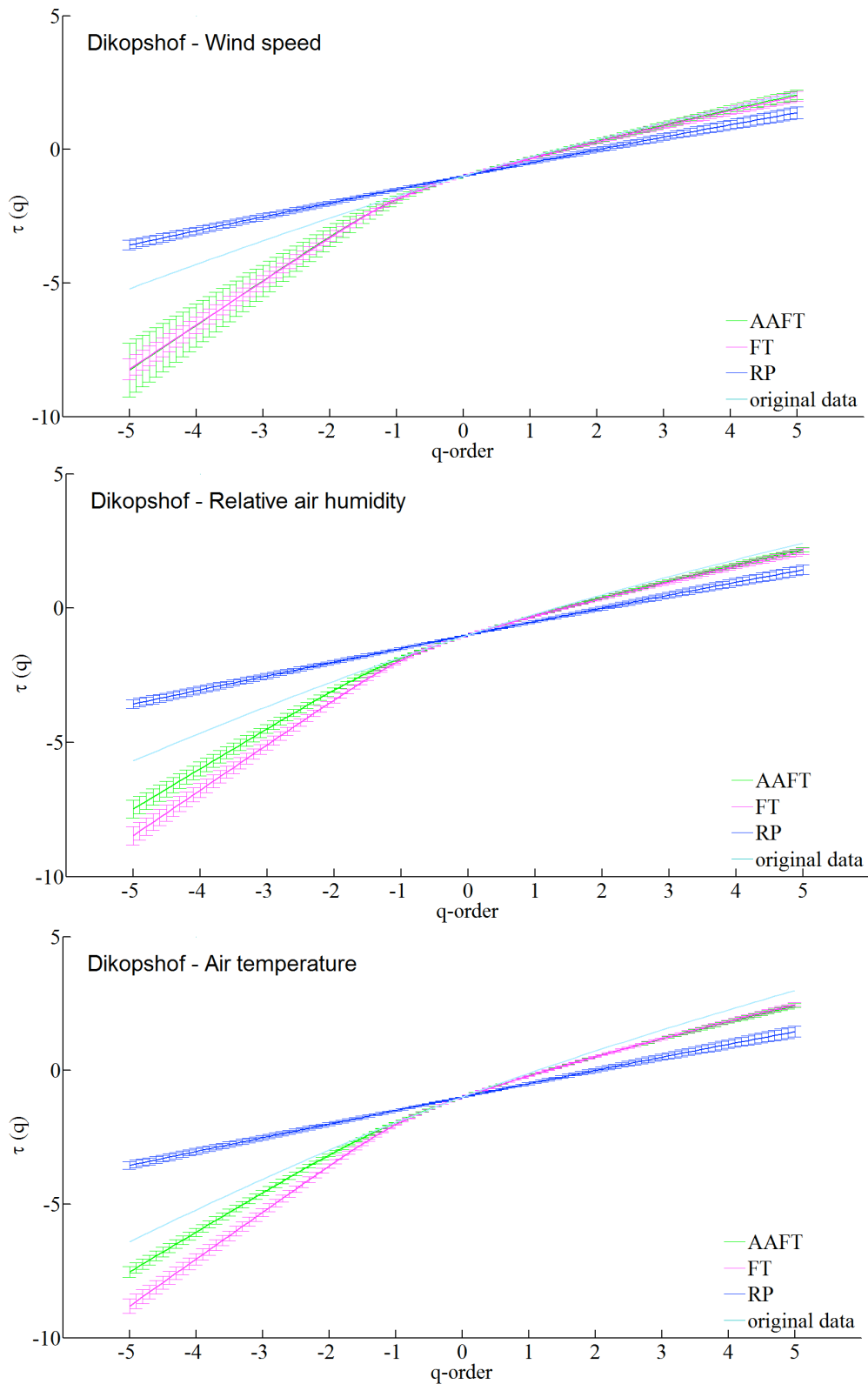
**Figure S1.** Multifractal spectra of FT randomized surrogate series

When comparing FT spectra (Fig. S1) and AAFT spectra (Fig.4 in the manuscript) with MFDFA spectra of original data (Fig. 2 in the manuscript), one can see that the multifractality of the original data is generally better preserved in the AAFT case. FT spectra have a similar shape in all cases, thus losing information (asymmetry, width).

To better check whether the multifractal spectra of the data significantly differ from FT and AAFT surrogates the analysis similar to the one presented in Palus (2008) was performed. Palus analysed broad range of  $q$  values (-10;10) and his multifractals were synthetic multifractals without influence of both long range correlations and broad tails in probability density function. Palus (2008) stated that in the range of  $q$  (-3;6) scaling exponents values are very similar for FT and IAAFT surrogates. Therefore, for the range of  $q$  (-5;5), which was used in our manuscript and is used frequently in many studies regarding multifractality, the largest deviation from original data should be seen for high

negative values of  $q$ . In the exemplary plots in Fig.S2, where scaling exponent  $\tau(q)$  for Dikopshof station for all meteorological time series is presented, the FT, AAFT and also randomised permutation (RP) surrogates are compared with the original data. The mean values (solid lines) and standard deviation bars of 100 randomly generated realizations of these transforms were plotted. Indeed, it is seen from these plots that for high negative values of  $q$  neither FT, nor AAFT preserve multifractality of original data (in agreement with Palus (2008)). It is also seen in this plot that RP destroys multifractality of the original data (straight line indicating a monofractal). In the case of precipitation, the scaling exponents of FT surrogates are closer to scaling exponents of original data, in other cases (global radiation, relative air humidity, air temperature) it is the opposite. Plots in Fig.S1 and Fig.S2 give evidence that multifractality of the time series studied in our manuscript is not just an artefact.





**Figure S2.** The scaling function  $\tau(q)$  for original data of Dikopshof time series (light blue line) and the mean of 100 realizations of AAFT (green solid line), FT (pink solid line) and RP (dark blue solid line) surrogate data. Vertical bars indicate 95% of distribution of appropriate surrogates.

## References

- Hall CM (2014) Complexity signatures for short time scales in the atmosphere above Adventdalen, Svalbard. *J Geophys Res Atmos* 119: 652–662, doi:10.1002/2013JD020988
- Mali P (2014) Multifractal characterization of global temperature anomalies. *Theor Appl Climatol*: 1–8, doi:10.1007/s00704-014-1268-y
- Min L, Shuang-Xi Y, Gang Z, Gang W (2013) Multifractal Detrended Fluctuation Analysis of Interevent Time Series in a Modified OFC Model. *Commun Theor Phys* 59: 1–6
- Nagarajan R (2007) Surrogate analysis of volatility series from long-range correlated noise. *Phys. A: Statistical Mechanics and its Applications* 374: 281–288.
- Palus M (2008) Bootstrapping multifractals: Surrogate data from random cascades on wavelet dyadic trees. *Phys Rev Lett* 101(13): 134101
- Schreiber T, Schmitz A (1996) Improved surrogate data for nonlinearity tests. *Phys Rev Lett* 77: 635–638
- Schreiber T, Schmitz A (2000) Surrogate time series. *Phys D* 142: 346–382
- Theiler J, Eubank S, Longtin A, Galdrikian B, Farmer JD (1992) Testing for nonlinearity in time series: the method of surrogate data. *Phys D* 85: 77