*The following supplement accompanies the article*

# Coral reef species assemblages are associated with ambient soundscapes

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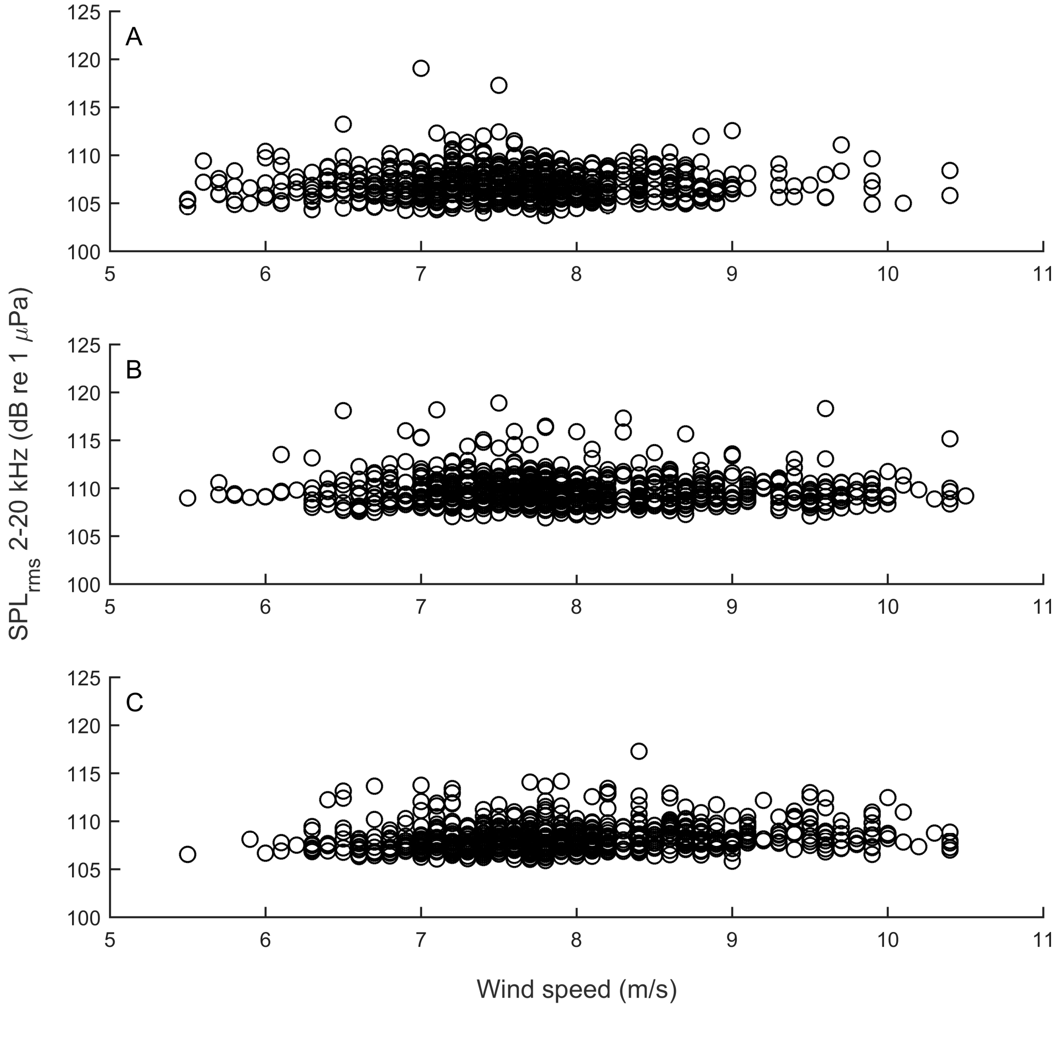
**Supplement.**

Fig. S1. Relationship between sound pressure level in the high frequency shrimp band (2-20 kHz) and wind speed for three US Virgin Islands reefs, Tektite (A), Yawzi (B), and Ram Head (C). Only Ram Head demonstrated a significant relationship between wind speed and sound pressure level, albeit weakly.



Here, we outline periodogram-based likelihood ratio (LR) tests for temporal and spatial non-stationarity of the spectral density function (sdf) of reef sound. The statistical results underlying these tests are available in Dzhaparidze (1986).

Let $X_{th}$, $t = 1, 2, \ldots, T$, be a collection of $T$ independent time series each of length $n$ recorded at times of day $h = 1, 2, \ldots, H$ on days $t = 1, 2, \ldots, T$ and let $f_{th}(\omega)$ be the unknown sdf at time of day $h$ on day $t$. Interest centers on testing the null hypothesis $H_o : f_{th}(\omega) = f_h(\omega)$ for all $h$ and $t$ that the sdf at each time of day is stationary over time against the general alternative hypothesis $H_1$ that it is not.

Let $I_{th}(\omega_j)$ be the periodogram ordinate for $X_{th}$ at Fourier frequency $\omega_j, j = 1, 2, \ldots, J$. It is a standard result that $I_{th}(\omega_j)$ is approximately independent of $I_{th}(\omega_k)$ and has an approximate exponential distribution with mean $f_{th}(\omega_j)$ and probability density function:

$$g\left(I_{th}\left(\omega_j\right)\right) = \frac{1}{f_{th}\left(\omega_j\right)} exp\left(-\frac{I_{th}\left(\omega_j\right)}{f_{th}\left(\omega_j\right)}\right)$$

The log likelihood is given by:

$$\log L = -\sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{j=1}^{J}\left(\log f_{th}\left(\omega_j\right) + \frac{I_{th}\left(\omega_j\right)}{f_{th}\left(\omega_j\right)}\right)$$

The maximum likelihood (ML) estimate of $f_{th}(\omega_j)$ under $H_1$ is simply $I_{th}(\omega_j)$ and the corresponding maximized value of the log likelihood is:

$$\log L_1 = -THJ - \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{j=1}^{J}\log I_{th}\left(\omega_j\right)$$

The ML estimate of the common sdf $f_h\left(\omega_j\right)$ under $H_o$ is the periodogram average:

$$\hat{f}_h\left(\omega_j\right)=\frac{1}{T}\sum_{t=1}^{T}I_{th}\left(\omega_j\right)$$

and the corresponding maximized value of the log likelihood is:

$$\log L_o=-THJ-T\sum_{h=1}^{H}\sum_{j=1}^{J}\log\hat{f}_h\left(\omega_j\right)$$

Finally, the LR statistic for testing $H_o$ against $H_1$ is:

$$LR=2\left(\log L_1-\log L_o\right)$$

which, under $H_o$, has an approximate chi squared distribution with degrees of freedom given by $\left(T-1\right)HJ$.

The same general approach can be used to test for spatial non-stationarity. Let $X_{tk}$, $t$ = 1, 2, …, $T$ be a collection of $T$ independent time series each of length $n$ recorded at locations $k$ = 1, 2, …, $K$ at times $t$ = 1, 2, …, $T$ and let $f_{tk}\left(\omega_j\right)$ be the unknown sdf at location $k$ at time $t$. Interest centers on testing the null hypothesis $H_o: f_{tk}\left(\omega_j\right)=c_{tk}\, f_t\left(\omega_j\right)$ that at each time the sdf's at the different locations are the same up to a multiplicative scaling against the general alternative hypothesis $H_1$ that they are not. For definiteness, under $H_o$, take $c_{t1}=1$ for all $t$.

As before, the maximized value of the log likelihood under $H_1$ is:

$$\log L_1=-TKJ-\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\log I_{tk}\left(\omega_j\right)$$

where $I_{tk}\left(\omega_j\right)$ is the value of the peridogram at time *t* and location *k* for Fourier frequency $\omega_j$. Maximizing the log likelihood under $H_o$ must be done numerically. In doing so, it is helpful to note that for fixed values of the scaling parameters $c_1 = 1, c_2, \ldots, c_K$, the ML estimate of $f_t\left(\omega_j\right)$ is the weighted average:

$$\hat{f}_t\left(\omega_j\right) = \frac{1}{K}\sum_{j=1}^{J}(I_{tk}\left(\omega_j\right)/c_k)$$

As before, the LR statistic for testing $H_o$ against $H_1$ is:

$$LR = 2\ (\log L_1 - \log L_o)$$

where $\log L_o$ is the numerically maximized log likelihood under $H_o$. Under $H_o$, *LR* has an approximate chi squared distribution with degrees of freedom given by $T\left(KJ - \left(K + J - 1\right)\right)$.

**REFERENCES**


Dzhaparidze KO (1986) Parameter estimation and hypothesis testing in spectral analysis of stationary time series. Springer-Verlag, New York