Supplementary Material for

"Role of mixotrophic nanoflagellates in the Eastern Mediterranean microbial food web"

E.Livanou, A. Oikonomou, S.Psarra, K. Lika

In this document we provide supplementary information for the Materials and Methods section and we give details on the mathematical formulation of the biogeochemical model. The rates of change in the state variables of the biogeochemical model are given by the differential equations presented in Table S1.

Transformations of substrates by the organisms are performed using the concept of Synthesizing Unit (SU) introduced by Kooijman (1998, 2010). SU can be described as generalized enzymes that bind arriving molecules of one or more substrates (e.g., food items, macromolecules or simple molecules) to form one or more products (e.g., macromolecules or biomass) (Kooijman 1998, Muller 2011). Arriving molecules bind to the SU according to their interaction (sequential or parallel) and their role in the product formation (substitutable or complementary) (Brandt et al. 2003, Kooijman 2010).

Pigmented nanoflagellates

Here we provide a brief description of the DEB model for a mixotrophic pigmented nanoflagellates (PNF) of Type IIA. The model for the autotrophic growth is presented in full detail in Livanou et al. (2019a) and the extension of the model to account for type IIA mixotrophy in PNF is presented in detail in Livanou et al. (2020). PNF biomass is described by 4 state variables corresponding to structure M_{VP} and three reserves masses M_i (here $i = E$ (generalized reserves), E_C (Carbon reserves), E_P (Phosphorus reserves)).

Table S2 summarizes the equations describing the metabolic fluxes in PNF. PNF are taking up inorganic carbon and phosphorus as well as bacteria (Eqs. S2.1 and S2.2 in Table S2). Photosynthetic units (PSU) harvest light (I) (Eq. S2.3) and produce reductant which is combined with inorganic carbon in a complementary and parallel transformation to form photosynthetically produced organic carbon (C) at rate j_{C,A_P} (Eq. S2.4). Carbon (C), inorganic phosphorus (IP) and phosphorus obtained from bacteria (B) are then assimilated (A) into generalized reserves (E) at a rate j_{E,A_P} (Eq. S2.5). The derivation of Eq. (S2.5) is based on the concept of SU and it is described in detail in Livanou et al. (2020). Briefly, in the case of Type IIA mixotrophy, organic carbon from photosynthesis and nutrients (either inorganic or nutrients from prey) are complementary substrates, meaning that both carbon and nutrients are needed to form generalized reserves. Following the standard notation of DEB theory θ represents a fraction of SU at a given state. A free SU, $\theta_{..}$, binds organic carbon from photosynthesis in parallel with inorganic phosphorus, giving the state θ_{CIP} to form generalized reserves and then return to the state $\theta_{...}$. Alternatively, a SU that has already bound organic carbon from photosynthesis, $\theta_{C.}$, may bind phosphorus from bacterial prey, giving the state θ_{CB} which then either produces generalized reserves and returns to the state $\theta_{..}$ or rejects it if inorganic phosphorus arrives, giving again θ_{CIP}. The rate of E-reserves assimilation (Eq. S2.5) is proportional to the fractions of SUs at quasi steady state $\hat{\theta}_{CIP}$ and $\hat{\theta}_{CB}$. The rates that organic carbon, bacteria, and inorganic phosphorus are used for E-reserves formation are given by Eq. S2.6, Eq. S2.7 and Eq. S2.8, respectively. All other physiological processes taking place in PNF are described mathematically in detail in Livanou et al. (2019a), with phosphorus being the limiting nutrient here.

PNF are the main producers of dissolved organic matter through two physiological mechanisms, one associated with growth (passive diffusion mechanism) (first term in Eqs. S2.16 and S2.17) and the other with the rejection flux of the surplus catabolic flux from C- and P- reserves due to stoichiometric constraints (active exudation mechanism) (Eq. S2.19). A detailed description of the DOM production by phytoplankton through these two excretion mechanisms is given in Livanou et al. (2019a). Moreover, PNF contribute to the labile DOM_L production via the excretion of the unassimilated organic carbon and phosphorus from the consumed bacteria (second term in Eqs. S2.16 and S2.17).

Heterotrophic bacteria

In the present model formulation bacterial biomass is assumed to have a constant stoichiometry. Table S3 summarizes the equations describing the metabolic fluxes in bacteria. Bacteria consume both labile and semi-labile dissolved organic matter, which is expressed in carbon DOC_x and phosphorus DOP_x, partitioned into two compartments comprising the labile ($x = L$) and semi-labile ($x = S$). The uptake of DOC_L and DOC_S is modeled based on the concept of SU, assuming that the two substrates are substitutable, meaning that the binding of one substrate disables the binding of the other and their processing by the SU is sequential, meaning that they can be separately assimilated into biomass (see Lika & Papadakis (2009) and Kooijman (2010) for details). The arrival rate of the two DOC substrates to the SU is given by Eq. S3.1 assuming a higher affinity (a_x) for the labile fraction, DOC_L. According to the rules of this transformation the uptake rates of DOC_L and DOC_S are given by Eq. S3.2 in Table S3 (see Lika & Papadakis (2009) and Kooijman (2010) for the derivation).

Both pools of DOM have a variable stoichiometry which is defined by the ratio $f_x = X_{DOP_x}/X_{DOC_x}$, $(x = L, S)$ and the uptake rate of DOP_x is determined by the uptake rate of DOC_x multiplied by the variable ratio f_x (Eq. S3.3). The rate at which DOM is assimilated is given by Eq. (S3.5) after taking into account the assimilation cost (quantified by the stoichiometric coefficients y_L and y_S) which is assumed to be higher for the semi-labile DOM. According to the stoichiometry of DOM, bacteria can either mineralize DOP into inorganic phosphorus or consume inorganic phosphorus additionally to DOP to cover their phosphorus requirements. The potential uptake rate of inorganic phosphorus is given by Michaelis-Menten kinetics (Eq. S3.4). Thus, the total available flux of phosphorus is $j_{IP,U_B} + j_{DOP,A_B}$ (Eq. S3.4 and Eq. S3.5). The actual carbon assimilation rate by bacteria j_{C,A_B} depends on the limiting element for biomass formation being phosphorus or carbon (Eq. S3.6). In the case where phosphorus is limiting then the extra carbon is respired whereas, when carbon is limiting then the extra phosphorus is mineralized (first term in Eq. S3.8).

The gross growth rate, j_{VB,G_B}, is given by Eq. (S3.7), after taking into account j_{VB,M_B} which is the maintenance rate. Maintenance results in the excretion of IC and IP quantified by the last term in Eq. (S3.8). Moreover, there are costs associated with growth and they result in the excretion of IC and IP quantified by j_{Ii,G_B} (second term in Eq. S3.8). Bacterial respiration is given by Eq. (S3.8). Bacterial Growth Efficiency (BGE) is calculated as the ratio between the net growth rate ($\dot{r}_B$) (Eq. S3.9) and the total organic carbon uptake rate ($j_{DOC_L,U_B} + j_{DOC_S,U_B}$) (Eq. S3.2), while Bacterial Production (BP) is calculated by multiplying the growth rate ($\dot{r}_B$) (Eq. S3.9) by the bacterial biomass (X_{VB}).

Heterotrophic nanoflagellates

In the present model formulation heterotrophic nanoflagellates (HNF) biomass is assumed to have a constant stoichiometry. Table S4 summarizes the equations describing the metabolic

fluxes in HNF. Heterotrophic nanoflagellates grazing rate on HB follows Michaelis-Menten kinetics (Eq. S4.1 in Table S4). The assimilation fluxes of bacterial biomass are obtained by assuming a constant assimilation efficiency of bacterial biomass y_{B_H} (Eq. S4.2). The unassimilated carbon and phosphorus are excreted as DOM_L (Eq. S4.3).

The gross growth rate, j_{VH,G_H}, is given by Eq. (S4.4), after taking into account the maintenance costs (j_{VH,M_H}). The mineralization fluxes for *IC* and *IP* are given by Eq. (S4.6) and Eq. (S4.7), respectively. The first term in Eq. (S4.6) and Eq. (S4.7) corresponds to losses due to growth overheads and the second term corresponds to costs of maintenance. The last term in Eq. (S4.7) quantifies the mineralization of extra phosphorus in the bacterial prey which is in excess of HNF phosphorus requirements.

Mortality

The mortality rates of HNF, PNF, and bacteria are given, respectively, by

$$j_{VH,D_H} = h_{H_{di}} + h_{H_{dd}} X_{VH} \tag{1}$$

$$j_{VP,D_P} = h_{P_{di}} + h_{P_{dd}} X_{VP} \tag{2}$$

$$j_{VB,D_B} = h_B \tag{3}$$

All organisms are assumed to experience a constant non-grazing mortality rate, which is proportional to structural biomass concentration (i.e., first term in Eqs. 1 - 3). A second rate which is density-dependent also contributes to total mortality experienced by heterotrophic (j_{VH,D_H}) and pigmented (j_{VP,D_P}) nanoflagellates (i.e., second term in Eqs. 1 - 2) and it is used to implicitly model predation effects of higher trophic levels on nanoflagellates.

Parameter values

Parameter values are presented in Table S5. Some parameter values were taken from published literature (see footnotes in Table S5). However, in the absence of an appropriate data set that would simultaneously cover all physiological processes of the specific groups of organisms studied here (e.g., nutrient acquisition, grazing rates, photosynthesis rate, growth rates, respiration rates etc.) other parameter values had to be assumed, however, when it was possible, a qualitative approach in parameter choice was taken. For example, the maximum structural mass specific uptake rate of inorganic phosphorus for HB ($j_{IP,U_{B_m}}$) was assumed to be 3 times that of PNF, an assumption of our model based on Vadstein (2000) who reports that heterotrophic bacteria have a maximum specific cell phosphorus-based uptake rate three times higher than green algae (a pigmented nanoflagellates group). The structural mass specific maximum uptake rate of phosphorus by PNF, $j_{IP,U_{P_m}}$ = 0.005 d^{-1}, was adjusted from the corresponding value of 0.007 d^{-1} obtained for *Pavlova lutheri* a small haptophycean marine nanoflagellate and it was calculated by dividing the maximum nitrogen specific uptake rate given in Marañón et al (2013) by 16 on the basis of the Redfield ratio (N:P = 16:1). The half-saturation constant for inorganic phosphorus of bacteria (KB_{IP}) was also assumed to be lower than that of PNF. This assumption reflects the fact that bacteria are better competitors for inorganic P uptake than eukaryotic phytoplankton at low inorganic P concentrations, as it has been demonstrated in many studies (e.g., Currie & Kalff 1984, Grover et al. 2000, Moutin et al. 2002). It should be noted here that that parameters related to nutrient uptake often depend on growth conditions, and thus can vary greatly in the literature (Litchman et al. 2007). The turnover time scales for labile and semi-labile DOM differ by an order of magnitude (Polimene et al. 2006). In out model this difference is reflected in the affinity constants of heterotrophic bacteria for labile and semi-labile DOM,

α_L and α_S, that were assumed to differ by an order of magnitude. Finally, it has been suggested that the assimilation of semi-labile polymeric DOM has a higher metabolic cost for bacteria due to the prerequisite of prior extracellular hydrolysis, resulting in lower growth yield (Middelboe & Søndergaard 1993). As such, the cost for assimilating semi-labile DOM, quantified by y_S, is taken to be higher than the cost for assimilating labile DOM, quantified by y_L, following the approach of Alekseenko et al. (2014). Since mixotrophs are assumed to have higher costs than pure autotrophs and heterotrophs (Raven 1997), the maintenance rate of HNF (j_{VP,M_H}) is taken to be lower than the one of PNF (j_{VH,M_P}). The maximum structural mass specific grazing rates of bacteria for HNF and PNF $j_{B,U_{P_m}}$ and $j_{B,U_{H_m}}$, respectively, were taken to be the same. This assumption finds support in some experimental evidence suggesting that the grazing rates of pure phagotrophs and phototrophs can be similar (e.g., Porter 1988, Tsai et al. 2011, Livanou et al. 2019b, Oikonomou et al. 2020). On the other hand, it has been been suggested that in low prey availability HNF cannot compete efficiently with PNF (Fischer et al. 2017). This is taken into account in the present model assuming a higher half-saturation constant for HNF in comparison to that of PNF.

Table S1: Model state variables. X_i is the concentration of compound i (in µmol L^{-1}) and j_{i,k_*} is the specific flux (in mol i (mol $V*)^{-1}$ d^{-1}) of compound i associated with process k and organism $*$, with $k \in \{U,A,R,C,Ex,D\}$[1], $* \in \{P,B,H\}$[2], and $i \in \{IC,IP,C,V*,E\,E_C,E_P,DOi_L,DOi_S,POi\}$[3]. j_{i,k_*} are given in Tables S2, S3 and S4 and in Eqs. 1-3.

No	Equation
S1.1	$\frac{d}{dt}X_{IC} = (-j_{C,A_P} + j_{IC,Ex_P})\, X_{VP} + j_{IC,Ex_B}\, X_{VB} + j_{IC,Ex_H}\, X_{VH}$
S1.2	$\frac{d}{dt}X_{IP} = (-j_{IP,U_P} + j_{IP,Ex_P})\, X_{VP} + j_{IP,Ex_B}\, X_{VB} + j_{IP,Ex_H}\, X_{VH}$
S1.3	$\frac{d}{dt}X_{VB} = (\dot{r}_B - j_{VB,D_B})\, X_{VB} - j_{B,U_H}\, X_{VH} - j_{B,A_P}\, X_{VP}$
S1.4	$\frac{d}{dt}X_{VH} = (\dot{r}_H - j_{VH,D_H})\, X_{VH}$
S1.5	$\frac{d}{dt}X_{VP} = (\dot{r}_P - j_{VP,D_P})\, X_{V_P}$
S1.6	$\frac{d}{dt}X_E = (j_{E,A_P} - j_{E,C_P})\, X_{VP} - j_{VP,D_P}\, X_E$
S1.7	$\frac{d}{dt}X_{E_C} = (j_{E_C,A_P} - j_{E_C,C_P} + \kappa_E\ j_{E_C,R_P})\, X_{VP} - j_{VP,D_P}\, X_{E_C}$
S1.8	$\frac{d}{dt}X_{E_P} = (j_{E_P,A_P} - j_{E_P,C_P} + \kappa_E\ j_{E_P,R_P})\, X_{VP} - j_{VP,D_P}\, X_{E_P}$
S1.9	$\frac{d}{dt}X_{DOi_L} = j_{DOi_L,Ex_P}\, X_{VP} + j_{VP,D_P}\, (n_{i,E}\, X_E + X_{E_i}) + (0.5\ j_{VH,D_H}\, n_{i,VH} + j_{DOi_L,Ex_H})\, X_{VH}$ $(-j_{DOi_L,U_B} + 0.5\ j_{VB,D_B}\, n_{i,VB})\, X_{VB} + d\ POi$, where $i = C,P$
S1.10	$\frac{d}{dt}X_{DOi_S} = j_{DOi_S,Ex_P}\, X_{VP} - j_{DOi_S,U_B}\, X_{VB}$, where $i = C,P$
S1.11	$\frac{d}{dt}X_{POi} = (0.5\ j_{VB,D_B}\, n_{i,VB})\, X_{VB} + j_{VP,D_P}\ \ n_{i,VP}\, X_{VP} + 0.5\ \ n_{i,VH}\ j_{VH,D_H}\, X_{VH} - d\ POi$, $(i = C,P)$

[1] U: uptake/consumption, A: assimilation, R: rejection, C: catabolism, Ex: excretion, D: death

[2] P: PNF, B: Bacteria, H: HNF.

[3] IC: inorganic carbon, IP: inorganic phosphorus,$V*$: structural mass of organism $*$, C:carbon, E generalized reserve of PNF, E_C: Carbon reserves of PNF , E_P: Phosphorus reserves of PNF, DOi_L, DOi_S: labile and semi-labile pools of dissolved organic matter in terms of carbon ($i = C$) and phosphorus ($i = P$), POi particulate organic matter pool in terms of carbon ($i = C$) and phosphorus ($i = P$).

Table S2: Equations that describe the physiological processes occurring in pigmented nanoflagellates (P). j_{i,k_P} is the specific flux (in mol i (mol VP)$^{-1}$ d^{-1}) of compound i^* associated with process k^{**} in P. Parameters explanation and values are given in Table S5.

No	Equation	Explanation
S2.1	$j_{IP,U_P} = j_{IP,U_{P_m}} \dfrac{X_{IP}}{X_{IP} + KP_{IP}}$	IP uptake
S2.2	$j_{i,U_P} = j_{i,U_{P_m}} \dfrac{X_i}{X_i + KP_i}, \quad (i = IC, B)$	Potential uptake/consumption
S2.3	$j_{I,U_P} = \rho_I \ \alpha_I I,$	Photons' arrival rate
S2.4	$j_{C,A_P} = \left(\dot{k}_C^{-1} + j'^{-1}_{IC,U_P} + j'^{-1}_{I,U_P} - (j'_{IC,U_P} + j'_{I,U_P})^{-1} \right)^{-1}$	C-formation rate[1]
S2.5	$j_{E,A_P} = \dot{k}_{CIP} \hat{\theta}_{CIP} + \dot{k}_{CB} \hat{\theta}_{CB}$	E- reserve formation[2]
S2.6	$j^+_{C,A_P} = y_{C,E} \ (\dot{k}_{CIP} \hat{\theta}_{CIP} + \dot{k}_{CB} \hat{\theta}_{CB})$	C consumption rate[2]
S2.7	$j_{B,A_P} = y_{B,E} \ \dot{k}_{CB} \ \hat{\theta}_{CB}$	Bacterial consumption rate[2]
S2.8	$j^+_{IP,A_P} = y_{IP,E} \ \dot{k}_{CIP} \hat{\theta}_{CIP}$	IP consumption[2]
S2.9	$j_{E_C,A_P} = j_{C,A_P} - j^+_{C,A_P}$	E_C-reserve formation
S2.10	$j_{E_P,A_P} = j_{IP,U_P} - j^+_{IP,A_P}$	E_P-reserve formation
S2.11	$j_{E_i,C_P} = m_{E_i}(\dot{k}_E - \dot{r}_P) \quad (i = -, C, P)$	Catabolic rate[3]
S2.12	$j_{E',C_P} = \left(\dot{k}^{-1}_{CIP} + j'^{-1}_{E_C,C_P} + j'^{-1}_{E_P,C_P} - (j'_{E_C,C_P} + j'_{E_P,C_P})^{-1} \right)^{-1}$	E$'$- reserve formation[4]
S2.13	$j_{VP,G_P} = y^{-1}_{E,VP}(j_{E,C_P} + j_{E',C_P} - j_{E,M_P})_+$	Gross growth rate
S2.14	$j^{M_P}_{VP} = \left(j_{E,M_P} - \min(j_{E,C_P} + j_{E',C_P}, j_{E,M_P}) \right) y^{-1}_{E,VP}$	Maintenance rate (structure)
S2.15	$\dot{r}_P = j_{VP,G_P} - j^{M_P}_{VP}$	Net specific growth rate
S2.16	$j_{DOC_L,Ex_P} = y_{DOC_L,VP} \ j_{VP,G_P} + j_{B,A_P}$	DOC_L excretion rate[5]
S2.17	$j_{DOP_L,Ex_P} = y_{DOP_L,VP} \ j_{VP,G_P} + (n_{P,VB} - n_{P,E} \ y^{-1}_{B,E}) j_{B,A_P} \quad (i = C, P)$	DOP_L excretion rate[5]
S2.18	$j_{E_i,R_P} = j_{E_i,C_P} - y_{i,E} j_{E',C_P} \quad (i = C, P)$	Rejection rate
S2.19	$j_{DOi_S,Ex_P} = (1 - \kappa_{E_i}) j_{E_i,R_P} \quad (i = C, P)$	DOC_S, DOP_S excretion rate
S2.20	$j_{IC,Ex_P} = (y_{C,E} - n_{C,E})(j_{E,A_P} + j_{E',C_P}) + j_{IC,M_P}$	IC excretion[6]
S2.21	$j_{IP,Ex_P} = j_{IP,M_P}$	IP excretion[7]

* I: irradiance, IC: inorganic carbon, IP: inorganic phosphorus, B bacteria, C: carbon, VP: structure, E: reserves, E_C: carbon reserves, E_P: phosphorus reserves, DOi_L and DOi_S: labile and semi-labile dissolved organic matter in terms of carbon ($i = C$) and phosphorus ($i = P$)

** U: uptake/consumption, A: assimilation, C: catabolism, M: maintenance, R: rejection, G: growth, Ex: excretion

[1] $j'_{IC,U_P} = j_{IC,U_P}/y_{IC,C}$ and $j'_{L,U_P} = j_{L,U_P}/y_{I,C}$

[2] $\hat{\theta}_{CIP} = \frac{j'_{IP,U_P}}{\dot{k}_{CIP}} \left(1 + \frac{j'_{C,A_P}}{j'_{IP,U_P} + j'_{B,U_P}} \left(1 + \frac{j'_{B,U_P}}{j'_{IP,U_P} + \dot{k}_{CB}} \right) \right) \hat{\theta}_{..}$ and $\hat{\theta}_{CB} = \frac{j'_{B,U_P}}{j'_{IP,U_P} + \dot{k}_{CB}} \frac{j'_{C,A_P}}{j'_{IP,U_P} + j'_{B,U_P}} \hat{\theta}_{..}$

with $\hat{\theta}_{..} = \left(\frac{j'_{IP,U_P}}{j'_{C,A_P}} + \left(1 + \frac{j'_{IP,U_P}}{\dot{k}_{CIP}} \right) \left(1 + \frac{j'_{C,A_P}}{j'_{IP,U_P} + j'_{B,U_P}} \left(1 + \frac{j'_{B,U_P}}{j'_{IP,U_P} + \dot{k}_{CB}} \right) \right) \right)^{-1}$

where $j'_A = \rho_A j_A / y_{A,Pr}$ stands for the rate at which substrate molecules, say of type A, arrive and bind at the SU, which is scaled to the number of molecules, $y_{A,Pr}$, required to produce one molecule of a product denoted as Pr. ρ_A is the probability that molecules of substrate A bind to the SU and it is taken to be 1 for all substrates.

[3] $m_E = X_E/X_{VP}$ and $m_{E_i} = X_{E_i}/X_{VP}$ with $i = C, P$ are the reserve densities.

[4] $j'_{E_C,C_P} = j_{E_C,C_P}/y_{C,E}$ and $j'_{E_P,C_P} = j_{E_P,C_P}/y_{IP,E}$

[5] $y_{DOi_L,VP} = y_{E,VP} n_{i,E} - n_{i,VP}$ (i = C, P)

[6] $j_{IC,M_P} = n_{C,E} j_{E,M_P} + j^{M_P}_{VP}(n_{C,VP} - y_{E,VP} n_{C,E})$

[7] $j_{IP,M_P} = n_{P,E} j_{E,M_P} + j^{M_P}_{VP}(n_{P,VP} - y_{E,VP} n_{P,E})$

Table S3: Equations that describe the physiological processes occurring in heterotrophic bacteria (B). j_{i,k_P} is the specific flux (in mol i (mol VB)$^{-1}$ d^{-1}) of compound i^* associated with process k^{**} in B. Parameters explanation and values are given in Table S5.

No	Equation	Explanation
S3.1	$j_{DOC_x} = \alpha_x \, X_{DOC_x}, \quad x = L,S$	DOC arrival rate
S3.2	$j_{DOC_x,U_B} = y_x \, j'_{DOC_x} \left(1 + \frac{j'_{DOC_L}}{k_L} + \frac{j'_{DOC_S}}{k_S}\right)^{-1}, \quad x = L,S$	DOC uptake rate[1]
S3.3	$j_{DOP_x,U_B} = f_x \, j_{DOC_x,U_B}, \; x = L,S$	DOP uptake rate[2]
S3.4	$j_{IP,U_B} = j_{IP,U_{B_m}} \, \frac{X_{IP}}{K_{IP,B}+X_{IP}}$	Potential IP uptake
S3.5	$j_{DOi,A_B} = j_{DOi_L,U_B}/y_L + j_{DOi_S,U_B}/y_S \; , i = C,P$	DOM assimilation rate
S3.6	$j_{C,A_B} = min((j_{DOP,A_B} + j_{IP,U_B})/n_{P,VB}, j_{DOC,A_B})$	C assimilation rate
S3.7	$j_{VB,G_B} = j_{C,A_B} - j_{VB,M_B}$	Gross growth rate
S3.8	$j_{Ii,Ex_B} = j_{Ii,A_B} + j_{Ii,G_B} + (1-1/y_L) \; j_{DOi_L,U_B} + (1-1/y_S) \; j_{DOi_S,U_B} + j_{VB,M_B} \, n_{i,VB}, \;\; i = C,P$	IC, IP excretion[3]
S3.9	$\dot{r}_B = j_{VB,G_B} \, y_{C,VB}^{-1}$	Net growth rate

* IC: inorganic carbon, IP: inorganic phosphorus, C: carbon, VB: structural biomass , DOi_L and DOi_S: labile and semi-labile dissolved organic matter in terms of carbon ($i = C$) and phosphorus ($i = P$)
** U: uptake, A: assimilation, M: maintenance, G: growth, Ex: excretion

[1] $j'_{DOC_x} = j_{DOC_x}/y_x, \;\; x = L,S.$

[2] $f_x = X_{DOP_x}/X_{DOC_x}, \;\; x = L,S$

[3] For $i = IP$, the term $j_{IP,A_B} = j_{DOP,A_B} - j_{C,A_B} \, n_{P,VB}$ can be either positive or negative, meaning that phosphorus is mineralized or taken up, respectively. For $i = IC$, the term $j_{IC,A_B} = j_{DOC,A_B} - j_{C,A_B}$ quantifies the excess carbon in DOM that cannot be assimilated and it is respired.
The term $j_{Ii,G_B} = y_{Ii,VB} \;\; j_{VB,G_B}, \;\; i = C,P$ where $y_{Ii,VB} = n_{i,VB}(1-(1/y_{C,VB}))$ (i = C, P) is the excretion flux of IC or IP during the synthesis of biomass VB, i.e. growth overheads.
The terms $(1-1/y_L) \; j_{DOi_L,U_B}$ and $(1-1/y_S) \; j_{DOi_S,U_B}$, with $i = C,P$ is the mineralized fraction of the uptake flux due to assimilation costs.
The last term represents the inorganic carbon and phosphorus released due to maintenance

Table S4: Equations that describe the physiological processes occurring in heterotrophic nanoflagellates (H). j_{i,k_P} is the specific flux (in mol i (mol VB)$^{-1}$ d^{-1}) of compound i^* associated with process k^{**} in H. Parameters explanation and values are given in Table S5.

No	Equation	Explanation
S4.1	$j_{B,U_H} = j_{B,U_{H_m}} \frac{X_{VB}}{X_{VB}+KH_B}$	Consumption of Bacteria
S4.2	$j_{C,A_H} = y_{B_H}\ j_{B,U_H}$	Carbon assimilation
S4.3	$j_{DOi_L,Ex_H} = (1-y_{B_H})\ n_{i,VB}\ j_{B,U_H}, \quad i = C,P$	DOM$_L$ excretion
S4.4	$j_{VH,G_H} = j_{C,A_H} - j_{VH,M_H}$	Gross growth rate
S4.5	$\dot{r}_H = j_{VH,G_H}\ y_{C,VH}^{-1}$	Net growth rate
S4.6	$j_{IC,Ex_H} = j_{IC,G_H} + n_{C,VH}\ j_{VH,M_H}$	IC excretion[1]
S4.7	$j_{IP,Ex_H} = j_{IP,G_H} + n_{P,VH}\ j_{VH,M_H} + (n_{P,VB} - n_{P,VH})\ j_{C,A_H}$	IP excretion[2]

* IC: inorganic carbon, IP: inorganic phosphorus, B bacteria, C: carbon, VH: structural biomass, DOi_L: labile dissolved organic matter in terms of carbon ($i = C$) and phosphorus ($i = P$)

** U: consumption, A: assimilation, M: maintenance, G: growth, Ex: excretion

[1] The term $j_{IC,G_H} = y_{IC,VH}\ j_{VH,G_H}$ where $y_{IC,VH} = (1-(1/y_{C,VH}))$ is the excretion flux of IC during the synthesis of biomass VH, i.e. growth overheads. The second term represents the carbon released due to maintenance.

[2] The term $j_{IP,G_H} = y_{IP,VH}\ j_{VH,G_H}$ where $y_{IP,VH} = (1-(1/y_{C,VH}))\ n_{P,VH}$ is the excretion flux of IP during the synthesis of biomass VH, i.e. growth overheads. The second term represents the phosphorus released due to maintenance. The last term represents the mineralization of extra P in the bacterial prey which is in excess of HNF P requirements.

Table S5: Table of parameter values. See Table S1 for explanation of symbols

Symbol	Value	Name (Units)
Pigmented Nanoflagellates		
ρ_{Imin}; ρ_{Imax}; b	0.2; 1; 2.84	For ρ_I: photons' binding probability (–) [1]
α_I	6×10^{-6}	Photons' arrival cross section (m^2 (μmol C)$^{-1}$) [1]
KP_{IC}	663	Half-saturation constant for IC (μM) [1]
KP_{IP}	0.05	Half-saturation constant for IP (μM)
KP_B	0.2	Half-saturation constant for B (μM)
$j_{IC,U_{P_m}}$	5.1	Max. uptake rate of IC (d^{-1}) [1]
$j_{B,U_{P_m}}$	0.7	Max. grazing rate of B (d^{-1})
$j_{IP,U_{Pm}}$	0.005	Max. uptake rate of IP (d^{-1})
$\dot{k}_C = \dot{k}_{CIP} = \dot{k}_{CB}$	1	Handling rates (d^{-1}) [1]
$\dot{k}_E$	1.5	Reserves turnover rate (d^{-1})
j_{E,M_P}	0.05	Maintenance rate (d^{-1}) [2]
$n_{C,E} = n_{C,VP}$	1	Chemical index of C in E, VP (mol C (mol C)$^{-1}$) [3]
$n_{P,E} = n_{P,VP}$	0.0094	Chemical index of P in E, VP (mol P (mol C)$^{-1}$) [3]
$y_{I,C}$	10	Stoichiometric coefficient (mol phot. (mol C) $^{-1}$) [1]
$y_{IC,C}$	1	Stoichiometric coefficient (mol IC (mol C)$^{-1}$) [1]
$y_{B,E}$	$n_{P,E}\ (n_{P,VB}\ y_{B_P})^{-1}$	Stoichiometric coefficient (mol C (mol C)$^{-1}$) [1]
y_{B_P}	0.9	Assimilated fraction of B [1]
$y_{C,E}$	1.2	Stoichiometric coefficient (mol C (mol C)$^{-1}$) [1]
$y_{IP,E}$	0.0094	Stoichiometric coefficient (mol P (mol C)$^{-1}$)
$y_{E,VP}$	1.2	Stoichiometric coefficient (mol C (mol C)$^{-1}$) [1]
$\kappa_{E_C} = \kappa_{E_P}$	0.5	Fraction of rejected flux returning to E_C/E_P (d^{-1})
$h_{P_{dd}}$	0.9	Density dependent mortality rate (d^{-1} μM $^{-1}$)
$h_{P_{di}}$	0.05	Mortality rate (d^{-1}) [2]
Heterotrophic Bacteria		
α_L	1	Affinity constant for DOM$_L$ (L (μmol C d)$^{-1}$)
α_S	0.1	Affinity constant for DOM$_S$ (L (μmol C d)$^{-1}$
y_L	1.5	Stoichiometric coefficient (mol C (mol C)$^{-1}$
y_S	3.3	Stoichiometric coefficient (mol C (mol C)$^{-1}$
$j_{IP,U_{B_m}}$	$3 * j_{IP,U_{P_m}}$	Max. uptake rate of IP (d^{-1})
KB_{IP}	0.01	Half-saturation constant for IP (μM)
$\dot{k}_L = \dot{k}_S$	1	handling rate of DOM$_L$, DOM$_S$ (d^{-1})
j_{VB,M_B}	0.01	Maintenance rate (d^{-1}) [2]
$n_{C,VB}$	1	Chemical index of C in VB (mol C (mol C)$^{-1}$) [4]
$n_{P,VB}$	0.02	Chemical index of P in VB (mol P (mol C)$^{-1}$) [4]
$y_{C,VB}$	1.2	Stoichiometric coefficient (mol C (mol C)$^{-1}$)
h_B	0.01	Mortality rate (d^{-1}) [5]
Heterotrophic Nanoflagellates		
KH_B	0.4	Half-saturation constant for B (μM)
$j_{B,U_{H_m}}$	0.7	Max. grazing rate of B (d^{-1})
y_{B_H}	0.95	Assimilated fraction of B
$y_{C,VH}$	1.1	Stoichiometric coefficient (mol C (mol C)$^{-1}$)
j_{VH,M_H}	0.01	Maintenance rate (d^{-1})
$n_{C,VH}$	1	Chemical index of C in VH (mol C (mol C)$^{-1}$) [3]
$n_{P,VH}$	0.0094	Chemical index of P in VH (mol P (mol C)$^{-1}$) [3]
$h_{H_{dd}}$	0.9	Density dependent mortality rate (d^{-1} μM^{-1})
$h_{H_{di}}$	0.05	Mortality rate (d^{-1}) [2]

[1](Livanou et al. 2020) and references therein (ρ_I is calculated according to Eq. (13) in Livanou et al. (2019a)), [2] Tsiaras et al. (2017), [3] On the basis of Redfield ratio (Redfield 1958), [4] Fagerbakke et al. (1996), [5] Allen et al. (2002)

LITERATURE CITED

Allen JI, Somerfield PJ, Siddorn J (2002) Primary and bacterial production in the Mediterranean Sea: A modelling study. J Mar Syst 33–34:473–495

Alekseenko E, Raybaud V, Espinasse B, Carlotti F, Queguiner B, Thouvenin B, Garreau P, Baklouti M (2014) Seasonal dynamics and stoichiometry of the planktonic community in the NW Mediterranean Sea: A 3D modeling approach. Ocean Dyn 64:179–207

Brandt BW, Van Leeuwen IM, Kooijman SALM (2003) A general model for multiple substrate biodegradation. Application to co-metabolism of structurally non-analogous compounds. Water Res 37:4843–4854

Currie D, Kalff J (1984) Can Bacteria Outcompete Phytoplankton for Phosphorus? A Chemostat Test. Microb Ecol 10:205–216

Fagerbakke KM, Heldal M, Norland S (1996) Content of carbon, nitrogen, oxygen, sulfur and phosphorus in native aquatic and cultured bacteria. Aquat Microb Ecol 10:15–27

Fischer R, Giebel HA, Hillebrand H, Ptacnik R (2017) Importance of mixotrophic bacterivory can be predicted by light and loss rates. Oikos 126:713–722

Grover JP (2000) Resource competition and community structure in aquatic microorganisms: Experimental studies of algae and bacteria along a gradient of organic carbon to inorganic phosphorus supply. J Plankton Res 22:1591–1610

Kooijman SALM (1998) The Synthesizing Unit as model for the stoichiometric fusion and branching of metabolic fluxes. Biophys Chem 73:179–188

Kooijman SALM (2010) Dynamic Energy Budget theory for metabolic organisation, 3rd Edition. Cambridge University Press, Cambridge

Lika K, Papadakis IA (2009) Modeling the biodegradation of phenolic compounds by microalgae. J Sea Res 62:135–146

Litchman E, Klausmeier CA., Schofield OM, Falkowski PG (2007) The role of functional traits and trade-offs in structuring phytoplankton communities: Scaling from cellular to ecosystem level. Ecol Lett 10:1170–1181

Livanou E, Lagaria A, Psarra S, Lika K (2019a). A DEB-based approach of modeling dissolved organic matter release by phytoplankton. J Sea Res 143:140–151

Livanou E, Lagaria A, Santi I, Mandalakis M, Pavlidou A, Lika K, Psarra S (2019b). Pigmented and heterotrophic nanoflagellates: Abundance and grazing on prokaryotic picoplankton in the ultra-oligotrophic Eastern Mediterranean Sea. Deep Sea Res II 164:100–111

Livanou E, Barsakis K, Psarra S, Lika K (2020) Modelling the nutritional strategies in mixotrophic nanoflagellates. Ecol Modell 428:109053

Marañón E, Cermeño P, López-Sandoval DC, Rodríguez-Ramos T, Sobrino C, Huete-Ortega M, Blanco JM, Rodríguez J (2013) Unimodal size scaling of phytoplankton growth and the size dependence of nutrient uptake and use. Ecol Lett 16:371–379

Middelboe M, Søndergaard M (1993) Bacterioplankton Growth Yield: Seasonal Variations and Coupling to Substrate Lability and β-Glucosidase Activity. Appl Environ Microbiol 59:3916–3921

Moutin T, Thingstad TF, Van Wambeke F, Marie D, Slawyk G, Raimbault P, Claustre H (2002) Does competition for nanomolar phosphate supply explain the predominance of the cyanobacterium *Synechococcus*? Limnol Oceanogr 47:1562–1567

Muller E (2011) Synthesizing units as modeling tool for photosynthesizing organisms with photoinhibition and nutrient limitation. Ecol Modell 222:637–644

Raven JA (1997) Phagotrophy in phototrophs. Limnol Oceanogr 42:198–205

Oikonomou A, Livanou E, Mandalakis M, Lagaria A, Psarra S (2020) Grazing effect of flagellates on bacteria in response to phosphate addition in the oligotrophic Cretan Sea, NE Mediterranean. FEMS Microbiol Ecol 96:fiaa086

Polimene L, Allen JI, Zavatarelli M (2006) Model of interactions between dissolved organic carbon and bacteria in marine systems. Aquat Microb Ecol 43:127–138

Porter KG (1988) Phagotrophic phytoflagellates in microbial food webs. Hydrobiologia 159:89–97

Redfield AC (1958) The biological control of chemical factors in the environment. Am Sci 46, 230A:205–221

Tsai AY, Gong GC, Sanders RW, Chen WH, Chao CF, Chiang KP (2011) Importance of bacterivory by pigmented and heterotrophic nanoflagellates during the warm season in a subtropical western Pacific coastal ecosystem. Aquat Microb Ecol 63:9–18

Tsiaras KP, Christodoulaki S, Petihakis G, Frangoulis C, Triantafyllou G (2017) Model simulations of a mesocosm experiment investigating the response of a Low Nutrient Low Chlorophyll (LNLC) marine ecosystem to atmospheric deposition events. Front Mar Sci 4:120

Vadstein O (2000) Heterotrophic, planktonic bacteria and cycling of phosphorus. In: Schink B (ed) Advances in microbial ecology. Springer, Boston, MA, p. 115–167